VOLUME 83 NO. SU2
NOVEMBER 1957

JOURNAL of the

Surveying and Mapping Division

PROCEEDINGS OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

BASIC REQUIREMENTS FOR MANUSCRIPTS

This Journal represents an effort by the Society to deliver information to the reader with the greatest possible speed. To this end the material herein has none of the usual editing required in more formal publications.

Original papers and discussions of current papers should be submitted to the Manager of Technical Publications, ASCE. The final date on which a discussion should reach the Society is given as a footnote with each paper. Those who are planning to submit material will expedite the review and publication procedures by complying with the following basic requirements:

1. Titles should have a length not exceeding 50 characters and spaces.

2. A 50-word summary should accompany the paper.

3. The manuscript (a ribbon copy and two copies) should be double-spaced on one side of 8½-in. by 11-in. paper. Papers that were originally prepared for oral presentation must be rewritten into the third person before being submitted.

4. The author's full name, Society membership grade, and footnote reference stating present employment should appear on the first page of the paper.

5. Mathematics are reproduced directly from the copy that is submitted. Because of this, it is necessary that capital letters be drawn, in black ink, ⅛-in. high (with all other symbols and characters in the proportions dictated by standard drafting practice) and that no line of mathematics be longer than 6½-in. Ribbon copies of typed equations may be used but they will be proportionately smaller in the printed version.

6. Tables should be typed (ribbon copies) on one side of 8½-in. by 11-in. paper within a 6½-in. by 10½-in. invisible frame. Small tables should be grouped within this frame. Specific reference and explanation should be made in the text for each table.

7. Illustrations should be drawn in black ink on one side of 8½-in. by 11-in. paper within an invisible frame that measures 6½-in. by 10½-in.; the caption should also be included within the frame. Because illustrations will be reduced to 69% of the original size, the capital letters should be ⅛-in. high. Photographs should be submitted as glossy prints in a size that is less than 6½-in. by 10½-in. Explanations and descriptions should be made within the text for each illustration.

8. Papers should average about 12,000 words in length and should be no longer than 18,000 words. As an approximation, each full page of typed text, table, or illustration is the equivalent of 300 words.

Further information concerning the preparation of technical papers is contained in the "Technical Publications Handbook" which can be obtained from the Society.

This Journal is published by the American Society of Civil Engineers. Publication office is at 2500 South State Street, Ann Arbor, Michigan. Editorial and General Offices are at 33 West 39 Street, New York 18, New York. $4.00 of a member's dues are applied as a subscription to this Journal.

AT,BD,CP,CO,HW,IR,SU,WW,PL.

Journal of the

SURVEYING AND MAPPING DIVISION

Proceedings of the American Society of Civil Engineers

CONTENTS

November, 1957

Papers

Number

Electric Analog for Level-Net Adjustment
by Hsuan-Loh Su 1443

Electronic Computers in Surveying Operations
by Arthur J. McNair 1444

Distance Measurement with the Geodimeter and Tellurometer
by John S. McCall 1445

Surveying and Mapping, St. Lawrence Power Project
by John D. Officer 1446

A Meteorological Method for Profile Surveying
by L. V. Toralballo 1447

Discussion 1448

Paper 1443 November, 1957 SU 2

Journal of the
SURVEYING AND MAPPING DIVISION
Proceedings of the American Society of Civil Engineers

ELECTRIC ANALOG FOR LEVEL-NET ADJUSTMENT

Hsuan-Loh Su[1]
(Proc. Paper 1443)

SYNOPSIS

The analogy between the electrical network and the level net suggests a method for adjustment of a level net. The electrical "computor" for this problem can be handled by civil engineers with ease. Calculation methods based on this principle are simple and results are automatically checked.

It will be shown in this article that analogy exists between the fundamental equations for solving a level net problem and those for an electrical network. Consequently, it is possible to solve a level net problem through the solution of a corresponding analogical electric network. The characteristics of such electric network are:

(1) Only resistance, current, and potential are involved in such network.
(2) All the resistances are known (The resistance in such network is equivalent to the reciprocal of the weight of the observed values).
(3) All the e.m.f.'s are known.
(4) The unknowns are the potential at joints and the currents.

For such a network some methods specially suitable for its solution can be devised. The direct approach is the experimental solution, to obtain the results directly from an electrical experiment. Such experiment is fortunately one of the simplest of its kind and not too difficult for a civil engineer or surveyor to handle. This type of experiment can give the correct result in much shorter time than any calculation particularly when the network involves quite a number of unknowns. The work of solving simultaneous equations will be more than doubled as the number of equations are doubled. But the volume of work involved in setting up an analogical electric network will be a linear function of the number of unknowns, i.e., the amount of work entailed for solving say 40 simultaneous equations, will be only 4 times greater, and no more, than solving 10 equations.

Note: Discussion open until April 1, 1958. Paper 1443 is part of the copyrighted Journal of the Surveying and Mapping Division of the American Society of Civil Engineers, Vol. 83, No. SU 2, November, 1957,

1. Engineer, Ovearup & Partners, London, England.

However, so far as the calculation methods are concerned the electric analogy still occupies an important place when it is combined with some methods of successive approximations. It is the purpose of this article to explore such methods. The principle underlying these methods will be briefly introduced and two different methods of successive approximations will be dealt with.

Notations

h_{AB}	THE OBSERVED VALUE OF DIFFERENCE OF ELEVATIONS BETWEEN STATION A AND B. POSITIVE WHEN A IS HIGHER THAN B.
h'_{AB}	THE APPROXIMATE DIFF. ELEV. COMPUTED FROM ASSUMED ELEVATIONS.
w_{AB}	THE WEIGHT OF h_{AB}.
H_A	THE ASSUMED APPROXIMATE ELEVATION OF STATION A.
x_A	THE CORRECTION TO THE APPROXIMATE ELEVATION OF A.
H'_A	THE MOST PROBABLE VALUE OF THE ELEVATION OF A. $H'_A = H_A + x_A$.
e_{AB}	THE DISCREPANCY BETWEEN THE DIFFERENCE OF APPROXIMATE ELEVATION AND THE OBSERVED DIFF. OF ELEV. $e_{AB} = H_A - H_B - h_{AB}$.
v_{AB}	THE RESIDUAL ERROR. $v_{AB} = H'_A - H'_B - h_{AB} = e_{AB} + x_A - x_B$.
E_{AB}	THE E.M.F. IN THE ANALOGIC CIRCUIT. POSITIVE WHEN DIRECTED FROM A TO B.
V_A	THE POTENTIAL AT A.
R_{AB}	THE RESISTANCE IN THE ANALOGIC CIRCUIT.
I_{AB}	THE CURRENT IN THE ANALOGIC CIRCUIT. POSITIVE WHEN FLOWING FROM A TO B.

Analogy Between Level Net and Electrical Network

By the theory of least square, the weighted sum of the square of the residual errors should be minimum. That is

$$\sum_{N=A}^{N} w_{PN} \cdot v_{PN}^2 = \text{MIN.}, \quad \frac{d}{d x_P}\left(\sum_{N=A}^{N} w_{PN} \, v_{PN}^2\right) = 0 \tag{1}$$

where Σ denotes a summation extending to all points directly connected with point P. By (1), we get

$$\sum_{N=A}^{N} w_{PN}\,(e_{PN} \quad x_P - x_N) = 0 \tag{2}$$

and by rearrangement, (2) becomes

$$x_p = -\frac{\sum(w_{PN}\ e_{PN})}{\sum(w_{PN})} + \frac{\sum(w_{PN}\ x_N)}{\sum(w_{PN})} \tag{3}$$

Now consider the portion of an electrical network. By Ohm's law, the following relation exists for the current along the path PN

$$I_{PN} = \frac{V_P - V_N + E_{PN}}{R_{PN}} \tag{4}$$

By Kirchoff's first law, the continuity of the flow of the currents requires that flow-in must equal to flow-out at any joint, i.e.,

$$\sum_{N=A}^{N} I_{PN} = 0 \tag{5}$$

Hence

$$V_p = -\frac{\sum \frac{E_{PN}}{R_{PN}}}{\sum \frac{1}{R_{PN}}} + \frac{\sum \frac{V_N}{R_{PN}}}{\sum \frac{1}{R_{PN}}} \tag{6}$$

The analogy between (3) and (6) can easily be seen. The equivalent entities are

$$V_p \sim x_P, \quad E_{PN} \sim e_{PN}, \quad \frac{1}{R_{PN}} \sim w_{PN}.$$

Problem for Illustration

The level net is shown in Fig. 1, and beside the surveyed line, its weight is marked.

A and D are two bench marks with an elevation of 28650 and 36720 respectively. The elevations of the bench marks are assumed to be accurate, and hence the potentials at A and D in the analogic electric network are naught. This means that joint A and D should be earthed. The surveyed results of the differences of elevations are collected in Table 1. h_{AB}, h_{CD}, h_{ED} and h_{FD} are chosen to compute the approximate elevations of B, C, E, and F, from which the approximate D.E's are obtained. and x are then calculated ($_{AB} = H_A - H_B - h_{AB}$).

The analogical electric network is shown in Fig. 2. Figures in brackets indicate the voltage of the electric cell, (or any electric source) which is numerically equal to e, the discrepancy. The direction of the electric current flowing out from the cell is determined with the sign of E_{MN}. If E_{MN} is positive, the direction of current is from point M to point N. Note $E_{MN} = -E_{NM}$.

Station (Assumed	B (30119)				C (35869)				E (34211)				F (33929)					
Elevation)	A	C	F	Sum	B	D	F	Sum	A	D	F	Sum	A	B	C	D	E	Sum
Weight of line (W)	1	1	2	4	1	1	2	4	1	1	2	4	2	2	2	2	2	10
Approx.D.E. computed from assumed elevation	+1469	-5750	-3810		+5750	-851	+1940		+5561	-2509	+282		+5279	+3810	-1940	-2791	-282	
Surveyed D.E. (h)	+1469	-5734	-3786		+5734	-851	+1947		+5556	-2509	+290		+5284	+3786	-1947	-2791	-290	
Discrepancy (e) = (approx. D.E.)-h	0	-16	-24		+16	0	-7		+5	0	-8		-5	+24	+7	0	+8	
(- we)		+16	+48	+64	-16		+14	-2	-5		+16	+11	+10	-48	-14		-16	-68

TABLE I. PRELIMINARY CALCULATIONS.

The value of the resistance is numerically equal to the reciprocal of the weight of the survey line, which is now represented by an electric path. The lines and the joints in the analogical electric network are exactly the same as in the original level net.

In Fig. 3A, the initial currents produced by the electric cells are shown along each line. In Fig. 4A, all arrows indicate the direction of the initial e.m.f. along several "closed" currents with the magnitude marked beside.

The detailed calculation will be explained in the following sections.

Method of Potential Adjustment

From Ohm's law,

$$V = IR \tag{1}$$

where R will always be constant in our problem since

$$R = \frac{1}{W} \tag{2}$$

while W, the weights of observations, is always known or assumed.

Therefore,

$$\Delta V = R\,\Delta I \text{ or } \Delta I = \frac{\Delta V}{R} \tag{3}$$

According to Kirchoff's first law, the continuity of the flow of currents requires

$$\Sigma\,(I + \Delta I) = 0 \tag{4}$$

where I is the assumed current, and ΔI is the amount of current in each line required to balance the flow-in and the flow-out at a joint. (The current flowing towards the joint is taken as negative and that away from the joint is positive).

Furthermore, a joint in the network can only have one value of potential, hence there can only be one ΔV.

$$\sum (I + \Delta I) = \sum I + \Delta V \sum \frac{1}{R} = 0$$

$$\Delta V = -\frac{\sum I}{\sum \frac{1}{R}} \tag{5}$$

It is obvious, from Eq. (5), that ΔV will be zero if the assumed currents are in equilibrium at this joint, i.e., $\Sigma I = 0$ is the criterion of a correct solution. When all ΔV become zero, V's need no more corrections and hence must be the correct values.

Procedure of Calculations

(1) Make a sketch of the level net as Fig. 3A, after making a table as Table I. This is also the analogic electrical network. It is advisable to draw such sketch on a tracing paper, which is strong enough to stand repeated erasion. It has been found best to sketch the network on the back side of the tracing paper so that it will not be removed when the figures are being erased.

(2) Compute the values of E, R etc., and mark them on the sketch, preferably on the back of the tracing paper.

(3) Compute the initial currents.

(4) Get the necessary potential adjustment ΔV from the algebraic summation of the initial currents at a joint $\Delta V = - \dfrac{\Sigma I}{\Sigma \frac{1}{R}}$

(5) Compute the currents induced by the increase of the potential ΔV.

(6) Treat other joints similarly until there is no unbalanced currents at any joint.

(7) Add the final values of the potential at joints, which are numerically equal to x, to the approximate elevation to obtain the most probable values.

Explanation of Calculation

The following paragraphs should be read in conjunction with Fig. 3A - 3F. (See Fig. 3A and 3B).

At B. It is apparent that the currents are not in equilibrium. The total flow-in (taken as negative) amounts to -64 units. To balance this, an amount of positive 64 units are required and this can be achieved by raising the potential at joint B by an amount calculated from Eq. (5), i.e.,

$$\Delta V = \frac{+64}{1+1+2} = +16$$

Owing to this imposed value of potential, the total flow-in is only 16 units along line FB and this is balanced by a total flow-out of 16 along BA. (See Fig. 3C).

At F. The unbalanced current amounts to

$$16 + 16 + 14 - 10 = +36 \text{ units.}$$

An equal amount of negative current is required to balance it. According to Eq. (5)

$$\Delta V = \frac{-36}{2+2+2+2+2} = -3.6$$

it is necessary to lower the potential, which is originally assumed to be zero, to -3.6 units.

Now the situation is as follows:

Along	FA	- 17.2
	FB	+ 8.8
	FC	+66.8
	FD	- 7.2
	FE	+ 8.8

These values give a total of zero. (See Fig. 3D).

At C. The amount of the accumulated current at joint E is -6.8. Therefore the potential has to be raised to 1.7.

$$\Delta V = \frac{6.8}{1+1+2} = +1.7$$

Then the current along CF will be -3.4 and along CB and CD, +1.7. The total is zero.
(See Fig. 3D).

At E. The total of flow-in is -8.8 and the flow-out is +5.0. Unbalanced current -8.8 + 5.0 = -3.8. Therefore, counter current required = +3.8. The potential is to be adjusted by an amount of $\frac{+3.8}{1 + 1 + 2} = 0.95$, say, 1.0. Therefore, potential = 0 + 1.0 = 1.0.
(See Fig. 3E).

At B. Owing to the adjustment done on joint F and C, the former balance of flow was upset. The amount of the unbalanced is now +16 - 8.8 - 1.7 = +5.5. The potential is to be adjusted so as to eliminate the +5.5 units. By Eq. (5)

$$\Delta V = \frac{5.5}{1 + 1 + 2} = -.14$$

Therefore, the potential of joint B becomes 16.0 + (-1.4) = +14.6. The situation is now:

BA	16 + (-1.4) = +14.6
BF	-8.8 + (-1.4/2) = -11.6
BC	-1.7 + (-1.4) = -3.1
Total	= -0.1 unit.

Such calculation as above can be continued until all ΔV's diminish to a certain value. The final values are shown in Fig. 3F.

Method of Current Adjustment

For a closed loop or closed circuit, the potential difference between a starting point and an end point (which may or may not be the same point) should be zero. This is Kirchoff's second law. In mathematical symbols, this statement will appear as follows:

$$\sum(V - E + \Delta V) = 0 \quad \text{or} \quad \sum(V - E) + \sum\Delta V = 0.$$

Since V = RI and V = R I, such relation can be written in a more useful form.

$$\sum(RI - E) + \sum R\Delta I = \sum(RI - E) + \Delta I \cdot \sum R = 0$$

$$\Delta I = \frac{-\sum(RI - E)}{\sum R}$$

It can be seen that ΔI will be zero when $\Sigma(RI - E) = 0$, which is the criterion of the "equilibrium" of the closed circuit.

Procedure of Calculation

The first few steps are similar to those in the method of potential adjustment, which has been discussed in the previous section.

After the initial unbalanced currents have been obtained the following steps are necessary:

(1) Choose several circuits such that each point at least is covered by two circuits. Number the chosen circuit.

(2) Calculate the "initial" e.m.f. along any circuit, and mark them in association with the number of circuit.

(3) Find a system of currents, which are "in equilibrium" at every joint. This can be achieved either by arbitrary assignment of certain values or, preferably, by a distribution of the initial unbalanced currents.

(4) Adjust the current in every section of a circuit by an amount which can be obtained through the circuit equation.

$$\Delta I = -\frac{\Sigma(RI - E)}{\Sigma R}$$

(5) Adjust each circuit in the same way as stated above until ΔI is sufficiently small to be neglected.

(6) The potential at a joint is now obtained and its numerical value, when added to the approximate elevation, gives the most.

The calculation is thus continued, and will only be introduced without explanations as follows:

Circuit	Σ RI	Σ E	ΣR	Δ I
A - F - D	1/2 x 10 - 1/2 x 68 = -29	5	1	$\frac{29 + 5}{1}$ = +34
A - E - D	-5 + 11 = 6	- 5	2	$\frac{6 + 5}{2}$ = -6
A - F - B - A	44x1/2 + 48x1/2 + 42 = 88	29	2	$\frac{-88 + 29}{2}$ = -30
B - C - F - B	18x1/2 + 6 - 14x1/2 = 8	1	2	$\frac{-8 + 1}{2}$ = -4
F - C - D - F	26 + 20 = 46	7	2	$\frac{-46 + 7}{2}$ = 120
A - F - E - A	7 + 5 + 11 = 23	18	2	-5/2 = -3
D - C - B - A	-2 + 12 = 10	16	3	-10 + 16/3 = +2
A - F - D	say 1	5	1	4
etc.				

CONCLUSION

These two calculation methods are alternatives of solving the normal equations which in our problem will be as follows:

$$\begin{aligned} 4x_B - x_C - 2x_F &= 64 \\ -x_B + 4x_C - 2x_F &= -2 \\ +4x_E - 2x_F &= +11 \\ -2x_B - 2x_C - 2x_E + 10x_F &= -68 \end{aligned}$$

The main advantage of these two methods is the automatic check contained in the methods in the process of solving the normal equations.

It should be emphasized that this article only introduces the basic idea of these methods, and that many other artifice can be devised and will be useful in solving certain problems.

The experimental method, itself, is merely to construct an electric computer, which in our case appears extraordinarily simple both to set up and to handle. There are numerous different computers available now. The author introduces this experimental method only because this particular type of computer can suit our purpose best in economy, simplicity, and efficiency.

FIG. 1

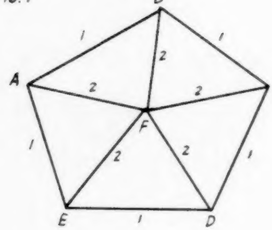


FIG. 2

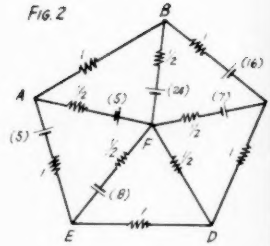


FIG. 3D

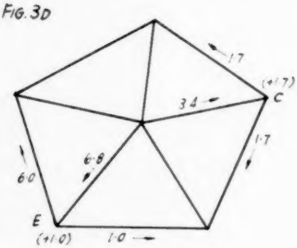


FIG. 3E

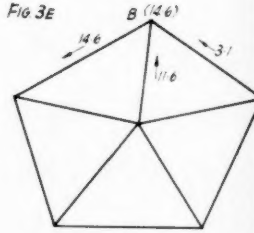


FIG. 4C

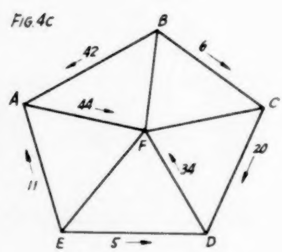


FIG. 4D

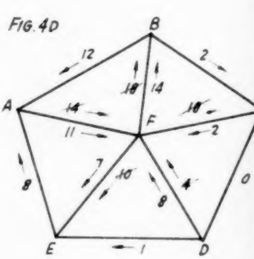


FIG. 3A

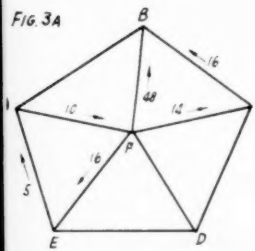


FIG. 3B

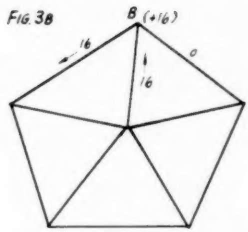


FIG. 3C

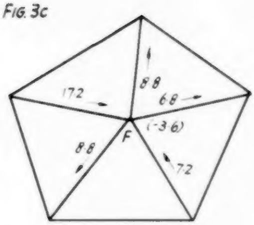


FIG. 3F

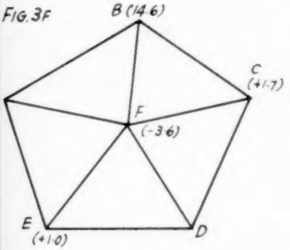


FIG. 4A

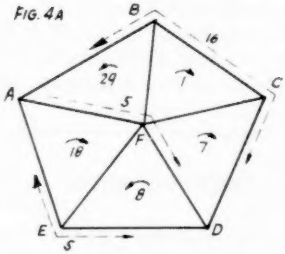


FIG. 4B

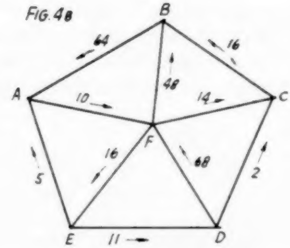


FIG. 4E

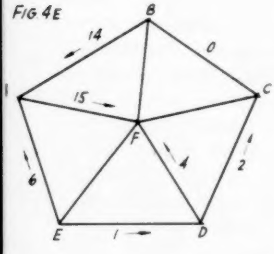


FIG. 4F

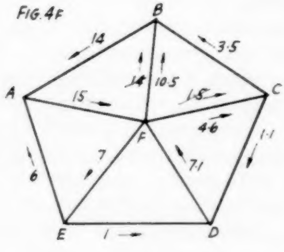
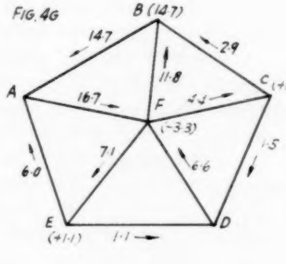


FIG. 4G



Journal of the
SURVEYING AND MAPPING DIVISION
Proceedings of the American Society of Civil Engineers

ELECTRONIC COMPUTERS IN SURVEYING OPERATIONS[a]

Arthur J. McNair,[1] M. ASCE
(Proc. Paper 1444)

SYNOPSIS

Electronic digital computers provide civil engineers with the means to compute two types of problems: (1) routine, repetitious, time-consuming problems, and (2) new, elaborate problems which are so staggering in magnitude that they have heretofore been beyond consideration. Examples of each type in surveying operations are given. Opportunities for new concepts in the teaching of surveying through use of an electronic computer are described.

INTRODUCTION

During the past two or three generations, the process of performing mathematical computations has advanced through several stages. From longhand—using pencil and paper and the procedure of casting out nines for check purposes—through the use of logarithms, sliderule, desk calculator, and now to electronic computers. The largest step, and certainly the one which is having the most rapid impact on the work of civil engineers, is the transition which is now being made from performing computations by the desk calculator to performing them on the electronic computer. Although engineers commonly are thought of as being one of the principal users of mathematics, it is interesting to note that engineers have been slow to take advantage of electronic computers. Accountants, statisticians, and research scientists have in general been in the forefront of the development and use of computing equipment. With the rapid development of electronic computers which is now going on and the fact that electronic computers are becoming readily accessible to almost any place in the United States, engineers are becoming more

Note: Discussion open until April 1, 1958. Paper 1444 is part of the copyrighted Journal of the Surveying and Mapping Division of the American Society of Civil Engineers, Vol. 83, No. SU 2, November, 1957.

a. Paper presented at a meeting of the American Society of Civil Engineers, Buffalo, N. Y., June 6, 1957.

1. Head of Surveying Department, Cornell University, Ithaca, New York.

familiar with electronic computing. They are losing their feeling of awe and mysticism and putting the machines to work.

Two Types of Computers

Most engineers are familiar with two types of computers; namely, digital and analog computers. The pencil and paper of longhand arithmetic and desk calculators are two types of digital computers. The sliderule, the Beggs' deformator, and the multiplex stereo-projector are typical analog computers. They are devices which set up an analogy which represents the quantities to be computed in a given problem. There are also both electrical and electronic analog computers. An excellent example of an electrical analog computer for surveying operations is the ESNA or Electrical Survey Net Adjuster developed by Julius Speert and others in the U. S. Geological Survey to represent the quantities which are to be adjusted in level nets. The McIlroy pipeline net analyzer is another example with which hydraulic engineers are familiar. But, there is one shortcoming of most analog computers, of which the slide-rule might be considered a most common example. They usually provide only the first two or three significant figures in a result, while the accuracy employed in surveying must usually be much better than one per cent. Because of the effect of cumulative errors, accuracies of at least 1:5000 are quite common; while accuracies of 1:1,000,000, especially in geodetic work, are not uncommon. No further reference will be made to analog computers.

Electronic Digital Computers

Hereafter reference to electronic computers should be understood to mean exclusively digital electronic computers. The manufacture of electronic calculators is going through a period somewhat akin to that experienced by the automobile in the early part of this century. Quite a number of special-purpose computers have been designed and built. Some of these computers are now orphans. Patent rights to others have been bought up by major companies. Several of the computer manufacturers are better known for their manufacture of aircraft and aviation instruments. Many of the electronic computers are designed for special purposes and are not particularly adapted to the problems encountered in civil engineering.

It is now necessary to draw a further distinction in electronic computers as it affects the ease with which an engineer can program his problems for computing and the speed at which the computing can be performed. This distinction has to do with the two types of programming, known as the interpretive language and the machine or basic language.

The language in which a general purpose computer is designed to accept its instructions is naturally designed for maximum efficiency and generality consistent with keeping the machine itself as simple as possible. This results in a language of instructions to the machine which is rather difficult for the uninitiated to use. This has been a deterrent to use of the machine for relatively short run problems. However, most of the better-known computers have now been programmed to simulate more specialized machines having instruction languages in which it is far easier to set up a restricted range of problems such as encountered in engineering computations. This makes it possible for the originator to put his own problem on a machine and to solve

it with a minimum of assistance from machine specialists, but at the expense of machine efficiency. This may result in a decrease of machine running speed by factors of up to 20, but for relatively short problems—say representing less than a man week of computation—the net effect is usually more efficient operation. Most electronic computers operate in what is known as fixed-decimal form with the decimal point at a predetermined position in the 10 or so significant figures available. However, in engineering it is usually necessary to operate with a specified number of significant figures with the total usually ranging beyond 10 figures. This requires careful scaling of a problem which is to be computed. Consequently, the machine is programmed to carry out a specified number of significant digits together with a tens exponent to indicate the order of magnitude of the quantity. A typical interpretive language code for an electronic computer might be: 1 add; 2 subtract; 3 multiply; 4 divide; 5 negative multiply; 300 extract square root; 303 calculate sine of an angle; 304 calculate cosine of an angle; etc. In a typical system approximately 50 such codes take care of most of the ordinary mathematical operations and elementary functions. Of course, others can be added to the repertoire in the "memory" as desired.

Survey Problems

It is only possible to indicate a few of the various problems encountered in surveying, which can profitably be computed through the use of electronic computers. It is certain that these computers are one more step in the trend to mechanization of human effort and relieving man of laborious and time-consuming operations. In general there are two types of problems to be encountered in surveying. One would be classified as the routine problems which have been almost the stock in trade of surveyors. The other might be classified in the category "Let your imagination go". In other words, the first and immediate use to which an electronic computer should be put is to relieve the burden of routine day-to-day computations. These are performed variously at the rate of from 5,000 to 500,000 mathematical operations per hour depending upon the type of computer. The second category of computations are those which have been so monstrous in magnitude that it has been unthinkable to even consider computing them until the advent of electronic computers. Many of these problems have been encountered in the general area of geodetic surveying. Frequently they require the solution of many simultaneous equations using up to ten significant figures. An example of each of the above two types of problems may suffice to indicate the scope of possibilities.

Of necessity, the examples are drawn from the experiences of the Surveying Department at Cornell University. One of the routine types of problems containing many repetitious operations is that of computing latitudes and departures of traverse lines and computing the coordinates of traverse points therefrom. At the Cornell Summer Survey Camp, which all civil engineering students attend, the field parties run a secondary control traverse consisting of 25 to 50 sides. After the students have done a sufficient amount of computations by hand or by desk calculator to have the principles firmly in mind, they organize the data for electronic computation and then take the values to the Cornell Computing Center where they have an opportunity to see the data run and to learn, in a general way, how the machines operate. There the

students punch the few necessary cards for the input data and follow the computation through the machine. This consists of converting their traverses in azimuths and lengths to latitudes and departures which are added consecutively to the coordinates of an initial point. If the traverses close within the required limit—at present a ratio of 1 to 5,000 is programmed in the machine—then the machine will continue computations. It proceeds to make an adjustment according to any prescribed rule—such as the transit rule or the compass rule. Adjusted coordinates are computed and the machine then computes the reverse problem providing adjusted azimuths and lengths for each line in the traverse. The computer requires a total of approximately 3 seconds to compute and adjust each traverse line so that if for instance ten parties were to arrive at the computing center each with a 50 course traverse, all 500 of the traverse lines would be computed, adjusted, and the revised coordinates, azimuths, and lengths would be tabulated in approximately a half hour. These quantities would all be carried out to 8 significant figures. This amount of work would keep a good man on a desk calculator busy for nearly a month, and then there might still be some mistakes in the results.

One example of problems which are so "monstrous" that they are rarely undertaken occurs in photogrammetry. When an aerial photograph contains the images of a specified number of known ground control points, it is possible to recapture the position in space and the pointing of the camera at the instant of exposure. One method of doing this is the Church space resection and orientation problem. The students in photogrammetry may spend ten hours on the desk calculator familiarizing themselves with the program of space resection and orientation of an aerial photograph. One or two hours are then spent in class discussing machine programming. This is usually their first real introduction to the subject. The average student usually does very little actual coding. To completely code this problem would require at least five hours with some guidance. Of course once the result of this five-hour program is completed and tested, it will accurately solve any number of this type problem without further attention. At the computing center the students punch the four input cards which are required for resection and orientation and watch the machine compute the results in somewhat less than three minutes.

Another similar example is the term paper recently submitted by an undergraduate student. He made a study of the errors introduced into the computed position and orientation of a camera exposure station by differential film shrinkage. This study of the effect of errors in film shrinkage would have taken a prohibitive 400 hours on a desk calculator, as against a little over an hour on the electronic computer. Using machine language the computing time could have been reduced to about ten minutes. The material he presented would have been ample for a thesis for a degree master of science a very few years ago. Now the study is simply a term paper.

Electronic Computers in Teaching Surveying

In engineering education, in greatly abbreviated form, the goals are to educate a man and to teach him to think; thus, the training of technicians is not one of the goals. In teaching surveying no attempt is made to train slide-rule operators, handbook engineers, multiplex operators, or programmers for electronic computers. Formal courses in the use of sliderule, desk

calculators, or electronic computers are not required in a civil engineering curriculum; however, the opportunity is provided for an engineering student to learn the use of these tools. At present, courses in surveying and in statistics offer the best introduction to electronic computers. It is probable that in the near future structural courses will make more use of the classical structural design methods and electronic computers, as contrasted with the present use of relaxation methods typified by moment distribution.

More than thirty universities throughout the country now have electronic computers on the campus. Most of these have an established computing center. They have time available for both regular student use at little or no charge and for commercial computing at established rates. At these institutions a great number of graduate students and quite a few undergraduate students attend the instructional periods which are offered and which give sufficient information that a beginner using the interpretive routine can program any of the simpler problems. Staff members at these computing centers are available to check over programming and to assist with the more complicated programs. Of course, there are also expert programmers available to assist with programming any complicated problems, particularly those where it is more desirable to carry out the computation in machine language.

At the present moment the subject of electric computers is a very exciting subject with new developments and uses appearing continually. However, teachers of surveying must watch that they do not get carried away by this new tool and become technicians. It should be studied as a new and very powerful tool which must be handled wisely so that it will perform economically. The subject of surveying and the subject of teaching surveying is at least as exciting as the subject of new computers. This is especially true in these days of electro-optical distance measurements, automation in photogrammetry, the International Geophysical Year, guided missiles, and similar developments. Thirty years from now new tools for computing may be developed which are as far in advance of electronic computers as electronic computers now are in advance of desk calculators. The same will doubtless be true of theodolites and distance measuring devices as they are known today. Despite this, surveying—which basically is the process of making measurements on or near the earth's surface—will be at least as interesting thirty years from now as it is at the present moment. It may be that surveying will be a study of physical quantities at the bottom of the ocean or miles under the surface of the earth; or it may be measuring the moon or some planet; it may be that topographic engineers will still be trying to get the first complete topographic map of the United States; or surveying may still be a problem of settling disputes between two neighbors over a property boundary, or rectifying the poorly made land surveys of the 19th and the first half of the 20th centuries. But the fundamentals of surveying—that is the problems of engineering measurement including geodesy and the shape of the earth—will be unchanged even though the techniques and instruments may be so different as to be unrecognizable to us at present.

The problem then is to make the best use of electronic computers in teaching surveying. One of the very important values is the requirement that computations be thoroughly organized. Programming concepts are fundamental to all types of computers. With pencil and paper a student frequently gets careless habits of picking out numbers somewhat at random and operating on them. He neglects to analyze his problem completely and to organize his

computations in an orderly manner. More especially, he usually thinks only in terms of the particular one problem at hand without considering the limiting values of his parameters. Programming for electronic computers requires complete planning of the quantities and operations in the entire problem so as to take maximum advantage of repetitious occurrence of these operations. The programmer must also consider such things as the size and range of the quantities, number of significant figures required, and positive and negative limits. This is excellent discipline for engineering students. Incidentally, it is equally good for the instructors and for practicing engineers. Whether a student actually uses a computer or not, the discipline of programming is valuable. It emphasizes as nothing else will the importance of analyzing and organizing calculations before getting lost in arithmetic.

An interesting use of the electronic computer in teaching surveying is the flexibility which it gives the professor in assigning problems to his classes. For instance, with no malice aforethought he can assign different values for a problem to each student in the class, program the problem once himself, and compute the twenty or more individual solutions in five minutes. An example of this type of problem which is good for instructional purposes, though it is generally shunned by engineers in practice, is the three-point problem of resection. The resection problem in hydrography is commonly solved using a three-armed protractor or some other mechanical or graphical method. Analytical computation of this problem is somewhat tedious, though it is performed by the U. S. Coast and Geodetic Survey as a procedure for locating horizontal control. The average student takes at least two hours to calculate the coordinates of the unknown point. It requires a little more than an hour to program this problem and test it for an electronic computer. From that time forward, any number of three-point problems can be solved at any time, making use of that same program. For instance, one instructor assigned every student in an advanced surveying course different values for a resection problem. It required five and one-half minutes time for the computer to solve seventy-five of these problems and an additional three minutes to print the results so they could be distributed to the students after their own work had been corrected. It is not hard for an instructor to visualize that this provides an opportunity to make a large number of very interesting correlations with the student work.

CONCLUSION

In conclusion, it should be said that electronic computers as we now know them, or improved versions which are bigger and faster, are here to stay. They are an engineering triumph which can ease the tedious burden of engineering computations. Those engineers involved in surveying operations have an opportunity to make immediate use of electronic computers. Any problem which is currently being solved by digital operations should be examined carefully to determine the economics of electronic computing as compared with conventional computing. Small problems which are performed only occasionally may still best be done by conventional methods. However, repetitive problems such as traverse computations, or cut and fill computations on highways, or geometric design of highways can probably be performed cheaper, and certainly more accurately, by electronic computers, thereby freeing personnel for other important jobs. Computing centers, especially those at

universities are usually more than happy to assist engineers with an educational program suitable to their own office operations.

The horizons have widened tremendously and civil engineers should make use of the opportunity to plan bigger, and to attack more specialized jobs which will require more intricate computations. For instance, the wedding of automatic photogrammetry and electronic computers is just looming over the horizon. It offers many opportunities. Computation of many networks and adjustment of errors perhaps involving the use of least squares and the solution of 5, 10, 40, or even 400 simultaneous equations can be performed very rapidly. Such problems have been unthinkable by hand methods. Engineers should give their imaginations free reign to make optimum use of this new tool.

Paper 1445 November, 1957 SU 2

Journal of the
SURVEYING AND MAPPING DIVISION
Proceedings of the American Society of Civil Engineers

DISTANCE MEASUREMENT WITH THE GEODIMETER AND TELLUROMETER[a]

John S. McCall[1]
(Proc. Paper 1445)

SYNOPSIS

The practical use of the measurement of distances by electronic means can easily be seen. Some of the applications would include: The rapid measurement of lines of a triangulation net; traverse in rough country or over bodies of water; and coastal hydrograph surveys.

INTRODUCTION

For many years attempts have been made to find a practical method which would permit the direct measurement of a distance between two points with an accuracy corresponding to a maximum error of a few inches in several miles. The Tacheometric, Subtense and Stadia methods have been used in many instances, but because of their limitations of range and/or accuracy, they were not acceptable for most types of higher order survey work. Precise lengths, as of base lines, were determined by following standard measuring procedures with invar tapes. Selection of the location was always very important because of the difficulty in taping over unfavorable terrain. Towers had to be erected (in most cases) on the terminal points of the base in order to accomplish the angle measurements necessary to connect the base into the primary triangulation scheme.

It has always been the surveyor's dream to have an instrument that could measure from mountain-top to mountain-top; across large bodies of water, or over other areas which could not be measured with the tape. The dream is coming true.

Note: Discussion open until April 1, 1958. Paper 1445 is part of the copyrighted Journal of the Surveying and Mapping Division of the American Society of Civil Engineers, Vol. 83, No. SU 2, November, 1957.

a. Paper presented at a meeting of the American Society of Civil Engineers, Buffalo, N. Y., June, 1957.

1. Geodetic Headquarters, Army Map Service, Washington, D. C.

There are two new devices which have been developed to effect direct distance measurements under these conditions. One type of instrument is known as the "Geodimeter"—which is an electro-optical instrument employing modulated light waves. The other instrument is known as the "Tellurometer". This employs radio microwaves in the tencentimeter band.

Each instrument will be described separately and briefly and indication will be made as to how each is being used today in conjunction with the science of surveying.

Geodimeter

Description of the Instrument

Background.—The Geodimeter, which derives its name from the words "geodetic", "distance" and "meter", was developed by Dr. Erik Bergstrand, Geodesist of the Swedish Geographic Survey Office. The instrument was designed for the purpose of measuring distances, using the fundamental constant —the velocity of light.

Principles and Measuring Techniques.—The Model 1 geodimeter which is composed of three basic units—the electrical unit, the light conductor unit and the reflector unit will be described first. There are four models of the geodimeter, Models 2, 3 and 4, which will also be described.

Measurement of distance with the geodimeter is carried out indirectly by determining the time interval for a highly collimated light beam to travel from the transmitting optics of the instrument to a distant reflector and return to the receiving optics of the same instrument. Knowing the velocity of light, it is then possible to compute the distance. The transmitted light beam is modulated by two crossed polaroids and a Kerr cell at approximately 7-1/2 meters per 1/4 wave-length; in other words, the Kerr cell effectively acts as an extremely fast electronic shutter that opens and closes millions of times per second by the application of a crystal-controlled, high-frequency voltage. The random light from a small 20-watt bulb in the instrument travels first through a condenser lens. From here, it travels through one of the polaroids, is brought to focus between the electrodes of the Kerr cell, passes through the second polaroid, then through the objective lens of the transmitter. At this point, the light beam passes through a simple reflex system which directs it toward the distant reflector which is, in most cases, a bank of retrodirective prisms. The beam is then reflected back toward the geodimeter where it is picked up by the receiver optics and is directed to the cathode of a photoelectric cell which changes the light beam to a usable electrical current. The phase relationship between the received light beam and the transmitted light beam can be made to balance out by the adjustment of a special phase delay system which results in a zero reading on a null indicator. At this point dial readings are recorded. The phase angle is then changed by 180°, and the phase delay system is again adjusted to obtain a zero reading on the null indicator. Two more readings 180° out of phase are taken to complete one set of readings on the mirror. A deflector arm is then dropped into position in the front of the instrument which directs the light beam up into the light conductor unit. The light conductor unit, which is calibrated in centimeters, is adjusted, selecting a whole number of light conductors plus a portion of a variable light conductor, and readings are again taken in four phase positions to arrive at approximately the same values that were obtained from the

readings on the mirror. To arrive at a more exact determination, an additional set of readings is taken on the mirror and a second set of light conductor readings is taken at a slightly different setting of the light conductor unit which would bracket the mean of the mirror readings between a high and a low light conductor setting. A simple algebraic formula can then be utilized to reduce the readings to an actual distance which corresponds to the phase relationship between the transmitted light beam and the received light beam. If the overall distance to be measured is known within 7-1/2 meters, it is very simple to obtain the exact distance. The overall distance is divided by the unit length of the frequency involved, this unit length having been determined during calibration procedure at the base plant. The number of unit lengths, less the remainder, is then multiplied by the unit length of the frequency involved. To this is added the distance corresponding to the phase relationship, then corrections are made for temperature, pressure and humidity, resulting in the slope distance of the line. If the distance is not known within 7-1/2 meters, but is known within one mile, the distance may be determined by taking readings on a second frequency. Using a rather lengthy formula, the exact slope distance may then be determined.

Weight and Means of Transport.—The three units of the Model I geodimeter in their cases weigh a total of approximately 450 pounds, making it rather difficult to transport the instrument over exceedingly rough terrain. However, in order to accomplish assigned missions, the geodimeter has been transported by airplane, helicopter, raft and truck, and, in several instances, the instrument has been removed from the cases and taken apart to be carried by back-pack to the station.

Accuracy.—The instrument is stated to have an accuracy in excess of 1 part in 250,000 with a maximum range of 30 to 40 miles. In 1953, measurements were made on two precise base lines in England. In each case the accuracy exceeded one part in 400,000.

Experience.—The value of the geodimeter has been proven many times. In Guadaloupe, a base line was measured over a sugar cane plantation. If the base had been measured by conventional methods the property damage alone would have exceeded $40,000. The total cost of the job, including transportation from Washington, D. C., and return, totaled slightly over $2,500. In southern Thailand, a 14 kilometer base line was measured across a bay which would have been impossible to measure by conventional means. In South America, one end of a geodimeter base line is at an altitude of over 14,000 feet.

Model 2 Geodimeter

Improvements over Previous Model.—Basically, the Model 2 geodimeter is the same as the Model 1, with the exception of an improved photocell circuit and the addition of a third frequency. The improved photocell circuit makes the adjustment of the receiver optics a little less critical, while the addition of the third frequency enables the operator to make a slightly more exact measurement of the distance.

Experience.—The Army Map Service does not have a Model 2 geodimeter, but has been called upon by another agency to repair and calibrate its instrument. Personnel from Army Map Service have had occasion to operate the Model 2 when working on a project in conjunction with another agency. It is our belief that the Model 2 is an improvement over the Model 1, but, as stated previously, they are basically the same.

Model 3 Geodimeter

Description of the Instrument.—The Model 3 geodimeter, as stated previously, works on practically the same principle as Models 1 and 2. However, it is composed of only two units, the light conductor unit being incorporated as an integral part of the electrical unit. The same reflector unit is used with the Model 3 as is used with Models 1 and 2.

Range and Accuracy.—This geodimeter, less the reflector unit and cases, weighs only 50 pounds. On the other hand, the accuracy has been reduced to approximately 1 part in 100,000 and the maximum range has been reduced to 20 miles.

Model 4 Geodimeter

Description of Instrument, Range and Accuracy.—The Model 4 Geodimeter is an extremely light-weight, portable instrument, weighing slightly over 25 pounds. Very little is known about this instrument,because it has not yet gone into production. A pilot model was shown recently at the American Congress on Surveying and Mapping meeting where it was learned that the distance readings are taken directly, thereby eliminating the need for a light conductor unit. The range of the instrument is approximately 3,000 meters with an accuracy of approximately 1 part in 5,000.

Tellurometer

Introduction

In 1954, Colonel H. A. Baumann, Director of Trigonometrical Survey of South Africa requested the Telecommunication Research Laboratory of South Africa to undertake the investigation of the use of radio waves for the measurement of the distance between two points to an accuracy acceptable for geodetic survey. The investigation was conducted by Mr. T. L. Wadley, who then developed an instrument known as the Tellurometer that operated in the 10 cm. wave-length (microwave) region and measured the travel time of radio waves over the length to be determined with an accuracy of a fraction of a millimicrosecond. The measurements were made by day and by night, visibility was immaterial. The instrumental technique gave the required degree of accuracy with the minimum of complexity. The instrument was lightweight, small and fully portable. In the early part of 1956 this new instrument was officially known as a new aid to the surveying profession. The first instrument arrived in the United States in March 1957 to be on display at the meeting of the American Society of Photogrammetry and the American Congress on Surveying and Mapping.

Description of the Instrument

The measurements are made between two instruments referred to as the Master and Remote stations, respectively. The measurements are made at the Master station, and an operator is required at the Remote station to perform the various switching operations on instructions from the Master observer. The Tellurometer instrument (either Master or Remote station) weigh about 24 pounds. The carrying case, power pack, tripod and battery (standard 6 volt, 40 amp. hours) weigh an additional 51 pounds. The power supply is

about 8 amps at 6 volts. The instrument is in a self-contained unit with built-in aerial system.

Principles and Measuring Technique

A continuous radio wave of 10 cm wave-length (3,000 Mc/sec) is transmitted from the Master station. This is modulated by a pattern frequency, which is 10 Mc/sec and other frequencies of similar order. The modulated wave is received at the Remote station, and in effect, re-transmitted to the Master station. The return wave is received back at the Master station is compared with the transmitted wave, the instrument indicating the phase shift between the outgoing and incoming modulation. The phase is indicated on an oscilloscope in the form of a circular sweep, or trace, in which a small break marks the phase against a circular scale. A decimal scale with 10 major and 100 minor divisions is used—the leading edge of the break in a clockwise direction being read usually to the nearest minor division. With a pattern frequency of 10 Mc/sec a complete rotation of the phase indication represents a change of 1/10 of a microsecond in the transit time over the double path. Each minor scale division thus represents one millimocrosecond and is equivalent to just under 6 inches. The 10 Mc/sec pattern, or A pattern phase, thus indicates the final two figures in the transit times in millimicroseconds. The unknown preceding figures, or whole number of A pattern rotations, are resolved by providing three further patterns (B, C, D). From the difference between the A pattern reading and the B, C, and D readings, coarse patterns are derived from which the preceding figures are determined. Four fine A readings are taken to increase accuracy. A reversal of phase provides reverse indications on both the +A and -A readings and, by averaging, eccentricity errors of the cathode display are eliminated, similar to averaging the circle readings on a theodolite.

Operating Procedure

Both operators align their beams to within 5 or 10 degrees of the true line. The Remote operator remains on a fixed carrier frequency, and it is the function of the Master station to find him. After the necessary adjustment of the display circle for shape, centering, focus and brightness, the coarse readings are made by the Master observer. On completion of the coarse readings, the Master observer will instruct the Remote operator to observe the meteorological conditions, pressure, temperature and humidity. These readings are recorded together at the Master station and give the mean meteorological condition at the start of the fine readings. The observer then notes the crystal temperature and calls for all fine A readings in turn, these being under control of the Remote operator only. For the next set of readings the observer will then instruct the Remote operator to shift frequency. The observer follows the frequency shift and completes the fine tuning. The observer then calls for all fine A readings again and so on until sufficient readings to suit conditions have been obtained. At the end the observer will again call for the meteorological conditions and after re-measuring crystal temperature and repeating his own meteorological measurements, the measurement is complete.

Experience and Accuracy

The equipment has been used in a number of field trials on geodetic nets

in South Africa, England and just recently in this country. Preliminary reports by the manufacturer state that the accuracy is somewhere in the neighborhood of 3 parts in 1 million plus or minus 2 inches. The distances have varied from 5 to 35 miles and a wide range of visibilities have been encountered. The equipment has shown its ruggedness by its ability to withstand being transported by various means. The Engineer Research & Development Laboratories have been running field tests for the past two months and the accuracies obtained are very encouraging. It has been noticed on certain lines that ground reflections have been encountered. The effects can be easily detected. Such effects are quite small in the majority of cases and are often considered negligible when a high degree of accuracy is not required.

CONCLUSIONS

1. Both the Geodimeter and Tellurometer depend on the fundamental constant of the velocity of propagation of electro-magnetic waves in vacuo. Reliable determinations were recently made by Essen, Bergstrand and Froome, and the General Assembly of the International Scientific Radio Union in 1954 accepted a value of 299,792.0 $\pm$ 2 Km/sec. A Tellurometer determination of the velocity of propagation using a geodetically established base line in South Africa gave a value of 299,792.9 Km/sec which is in very good agreement with the previously determined values.

2. The practical use of the measurement of distances by electronic means, such as the Geodimeter and the Tellurometer, in solving some of the surveyor's problems, can easily be seen. Some of the applications would include:

   a. The rapid measurement of lines of a triangulation net without recourse to the time-consuming and expensive measurement of bases and the subsequent expansion into the main triangulation scheme with accompanying loss of accuracy.

   b. Traverse in rough country or over bodies of water where conditions are not suitable for direct distance measurement.

   c. Coastal hydrographic surveys by intersection, or trilateration.

   d. Supplemental control for aerial photography, such as the establishment of picture points. These points could be fixed by intersection methods.

3. It may thus be expected that the continuing development of electromagnetic radiation and instruments will further revolutionize the traditional surveying methods, and the surveyor's tape may become obsolete in the not too distant future.

Paper 1446 November, 1957 SU 2

Journal of the
SURVEYING AND MAPPING DIVISION
Proceedings of the American Society of Civil Engineers

SURVEYING AND MAPPING, ST. LAWRENCE POWER PROJECT[a]

John D. Officer,[1] A.M. ASCE
(Proc. Paper 1446)

ABSTRACT

The agencies developing the St. Lawrence Power Project solved many surveying and mapping problems while obtaining detailed topographic and property maps and establishing horizontal and vertical control for structures. Each agency had a different method of accomplishing the same coordinated results.

SYNOPSIS

At the time of authorization of the St. Lawrence Power Project in 1954, the survey and mapping data available consisted of a network of first order horizontal and vertical control along both sides of the river, standard quadrangle topographic maps, larger scale topographic and property maps of part of the area, records of river soundings, aerial photographs, and records from the 1942 report of the Corps of Engineers, U. S. Army.

The basic desired data were: (a) detailed topography, both above and below water surface, for location and design of all features, and determination of the area to be flooded; (b) detailed property maps of all land to be acquired and (c) local horizontal and vertical control for all structures. All maps must be reference to a common datum by all agencies concerned with the Seaway and Power Projects.

The St. Lawrence Power Project is being developed as a joint venture by the Power Authority of the State of New York and the Hydro-Electric Power Commission of Ontario. The Power Authority retained the firm of Uhl, Hall

Note: Discussion open until April 1, 1958. Paper 1446 is part of the copyrighted Journal of the Surveying and Mapping Division of the American Society of Civil Engineers, Vol. 83, No. SU 2, November, 1957.

a. Paper presented at a meeting of the American Society of Civil Engineers, Buffalo, N. Y., June, 1957.

1. Construction Mngr., Uhl, Hall & Rich, Engrs. for the Power Authority of the State of New York, Massena, N. Y.

and Rich of Boston, Massachusetts, to do its engineering. Ontario Hydro does its own engineering.

The St. Lawrence Seaway is being developed by the St. Lawrence Seaway Development Corporation, with the Corps of Engineers, U. S. Army as its construction agent, and the St. Lawrence Seaway Authority of Canada.

Solution of the basic problem was accomplished by cooperation of all agencies employing their own forces, enlisting the aid of other government agencies and directly or indirectly obtaining services from private concerns.

The principal difficulties encountered in surveying were the fast current in the river, flat topography, dense forest growth and a multitude of swamps.

For mapping the results of surveys, the main difficulty was in discovering the discrepancies in existing maps and then reconciling the different methods used.

Control Surveys

A triangulation net covering the St. Lawrence River from Cape Vincent to St. Regis, New York, was made by the U. S. Lake Survey about 1872. This agency ran precise levels along the United States side of the river in 1898 and 1899. Vertical control is based on the 1903 and 1935 adjustments of these levels.

The Canadian Department of Transport during 1952 and 1953 ran a line of precise levels from Montreal to Kingston to tie in all gauges along its side of the river. The older gauges were renovated and some 40 new temporary gauges set in 1952. All were tied into the level line and to the Canadian Coordinate system. Ties were made between the Geodetic Survey of Canada datum and the U. S. Lake Survey datum at six points across the St. Lawrence between Cornwall and Kingston. Ties were made with reciprocal levels run with a Wilde N3 precise level. There was a small discrepancy between the two datums so it was decided to use the U. S. Lake Survey 1935 datum for the entire project.

In 1911 and 1912, the International Waterways Commission established its triangulation system covering the same area as that of the Lake Survey. The International Boundary Commission has a triangulation net to tie in the international boundary which generally is in the center of the main channel of the river.

In 1941 and 1942, the Corps of Engineers established secondary horizontal and vertical control for all its proposed power and seaway features. It also transferred all horizontal control points from geographic coordinates to the Transverse Mercator coordinates (east zone) for the State of New York, which had been designated as the basis of coordinates for the project. This was done by the method outlined in "Special Publication No. 193—Manual of Plane Coordinates Computations" as published by the U. S. Coast and Geodetic Survey.

The Frontier Corporation in 1907 completed a survey covering the area along the river between Ogden Island and Polly's Gut and extending inland for varying distances. This is commonly called the Ross survey. Maps prepared from this survey, revised in 1923, show topographic and property line features including bearings and distances. Mean sea level was used as the datum for these maps, which have a scale of 1 inch equals 200 feet; contour interval 5 feet plotted on an arbitrary plane coordinate system. The Corps of Engineers

made field ties from Ross triangulation stations to International Waterways Commission triangulation stations and an equation was computed between the Ross coordinates and the Transverse Mercator coordinates. This latter system was superimposed on the Ross maps.

Around 1924, Mr. M. C. Henry surveyed the area on the Canadian side from Prescott to St. Regis and from the St. Lawrence back to approximately the 250 ft. contour. In plotting the notes of this survey, plane coordinates were used, based on a meridian passing through International Boundary Monument No. 11. Apparently no correction was made for curvature of the earth so that, even though individual areas are well mapped, the overall survey does not check with the triangulation net.

During the summer and fall of 1954, the U. S. Lake Survey and Ontario Hydro checked numerous bench marks in the area and established new ones. Zero bars for river guages are rechecked regularly every six months.

In the late fall of 1952, Ontario Hydro, convinced that approval of the Power and Seaway Projects was imminent, moved a small survey party to the project to start on detailed hydraulic studies for construction of a model of the river. More men came in during the early spring of 1953 and proceeded with detailed localized control for sounding lines in those sections of the river where channel dredging would be required.

In the late fall of 1953, this localized control was tied into the main triangulation net and the main net rechecked. The primary net was held to an error of closure of 4 seconds. Secondary nets were held to 9 seconds.

During the many intervening years since this net had been established, vegetation had obscured sight lines. There being no authorization for the projects at this time, Ontario Hydro's surveyors did not have permission to clear lines and therefore operated under considerable difficulty.

In 1954, the Power Authority of the State of New York and the Hydro-Electric Power Commission of Ontario agreed upon a simple coordinate system of horizontal control and Ontario Hydro computed coordinates for all reference monuments. The coordinate system is tied to a meridian through I.B.M. No. 11 at the northeast corner of Barnhart Power Plant. The area covered by the system starts in the vicinity of Prescott, Ontario, and Ogdensburg, New York, and extends about forty-six miles down the river to the vicinity of Cornwall, Ontario, and St. Regis, New York. The width varies from one to three miles on each side of the river.

Topographic Surveys

In connection with the problem of obtaining detailed topographic maps for design and other purposes, an analysis was made of the available data. The standard quadrangle maps of the United States Geological Survey are on a scale of 1:62,500 and 1:25,000, contour interval 20 feet. The standard Canadian topographic maps are on a scale of 1:63,360, contour interval 25 feet. Other available topographic maps did not give complete coverage of the area. From these quadrangle maps, sufficient overall topography was obtained to determine the areas for which more detailed topography would be necessary.

The Power Authority, through its Engineer, Uhl, Hall & Rich contracted with LaFave Associates of Massena, New York, for land and aerial topographic mapping services and with Aero Service Corporation of Philadelphia, Pennsylvania, for aerial photographic and topographic mapping services. Maps were

made on a scale of 1 inch = 400 feet, contour interval 5 feet, for an area of approximately 61,000 acres and on a scale of 1 inch = 200 feet, contour interval 2 feet, for an area of approximately 6,700 acres.

Areas in which construction work had to be started immediately and for which some topographic information was already available were mapped by plane table on a scale of 1 inch = 400 feet contour interval 5 feet. Generally, these areas were relatively free of forests. From these data, it was possible to prepare designs and drawings of sufficient accuracy to permit the letting of contracts for major features such as Long Sault Dam, Barnhart Power Plant, and the Long Sault Dike.

Ontario Hydro with its own survey crews took detailed topography of town and structure sites. The areas were laid out on a 100-foot grid system by transit and chain and elevations were determined by levels. The contour interval was 1 foot or 2 feet depending on the detail required.

Ontario Hydro contracted with Spartan Air Service of Ottawa, Ontario, to photograph and plot contours for the areas around Barnhart, Long Sault and Iroquois on a scale of 1 inch = 100 feet and for an area of approximately 60,000 acres on a scale of 1 inch = 400 feet. Ground control was established by Ontario Hydro.

Right-of-Way surveys, computations, and other engineering services were obtained by contract from Erdman and Hosley of Rochester, New York. This group ran flow line contours and traverse in several areas to determine the limits of land to be taken in relation to the boundaries of individual ownerships. As soon as topographic maps were available from aerial surveys, all data thus obtained were plotted on the new maps.

Structures

Specifications for construction for Power Authority work provided for the contractors to furnish most survey work. Each major structure contract between the Power Authority and its contractors has in the General Conditions a paragraph similar to the one quoted below from St. Lawrence Contract No. 9 for the Construction of Barnhart Island Power Plant.

"2-07. <u>SURVEYS</u>

(a) The Engineer has established the following base line and bench mark at the site of the work:

(1) Control points on the centerline of units consist of a brass disc set in concrete at station 50 + 00 whose coordinate position is N92,993.08 and E164,929.04. A similar disc on the Canadian side at station 10 + 00 to indicate centerline direction is N52°18'59.5" E of station 50 + 00.00.

(2) Bench Mark No. 2713—Iron pipe with brass cap on Barnhart Island in St. Lawrence River, 2-1/2 miles upstream from Roosevelt Bridge in southeast corner of junction of first east-west stone fence from south shore and first north-south stone fence from east shore. It is approximately 590 feet right of station 48 + 25, centerline of units. Elevation (U.S.L.S. 1935 Adj.) 218.232.

(b) From the base line and bench mark established by the Engineer, the Contractor shall complete the layout of the work, and shall be responsible for all measurements that may be required for execution of the work to exact position and elevation as prescribed in the specifications, or shown on the

drawings, and subject to such modifications as the Engineer may require to meet changed conditions or as a result of necessary modifications to the contract work.

(c) The Contractor shall furnish at his own expense, such stakes, templates, platforms, equipment tools and materials, and all labor as may be required in laying out any part of the work from the base lines and bench marks established by the Engineer. If, for any reason these monuments are disturbed, it shall be the responsibility of the Contractor to re-establish them without cost to the Authority as directed by the Engineer. The Engineer may require that construction work be suspended at any time when location and limit marks established by the Contractor are not reasonably adequate to permit checking completed work, or the work in progress.

(d) The Contractor shall furnish, at his own expense, all personnel, equipment, and material required to make such surveys as are necessary to determine the quantities of work performed or in place. All original field notes, quantity computations, cross-sections, and other records taken by the Contractor, or required by the Engineer for the purpose of quantity surveys shall be furnished promptly to the Engineer at the site of the work. The field notes shall be taken on waterproof, loose-leaf transit and/or level note sheets satisfactory to the Engineer. Notes shall be properly identified by giving title, page number, date, weather, and the names and positions of all party personnel. Computations of volumes, records of embedded material, and other records of the work shall be properly identified by giving title, page number, date, and names of individuals performing and checking such computations or other data, and shall be furnished so as to be capable of duplication by contact printing methods. The original copy of field notes, computations, and records shall become the property of the Authority.

(e) Unless waived in each specific case, quantity surveys shall be made under the direction of the Engineer. The Authority reserves the right, despite such waiver, to conduct such check surveys as it may deem necessary. If errors are found in the Contractor's surveys, all expense of the check survey, together with the expense of such surveys as are necessary to rectify the errors or omissions, will be chargeable to the Contractor and deducted from monies due or to become due the Contractor.

(f) The Contractor shall prepare all estimates of partial payments due for work performed together with supporting data and computations as are deemed necessary by the Engineer to determine the accuracy of the estimate. The summary of estimates shall be submitted on the form prescribed by the Engineer. Failure of the Contractor to submit estimates of partial payments, or lack of accurate supporting data, shall be sufficient reason for withholding payment until such omissions or errors are rectified.

(g) All costs of surveys, computations, and assembly of data performed by the Contractor for layout of work and for determination of quantities and payment estimates as described herein will be considered incidental to and included in the prices and payments for the various items listed in the bid schedule."

The Contractor for Barnhart Power Plant, under the sponsorship of B. Perini & Sons, Inc. of Framingham, Massachusetts, started immediately upon award of contract to establish references for the centerline of units. The reference monuments on each end of the structure had been located by triangulation. Until the cofferdams were completed, there was no way to precisely

chain the distance between the two. This was done in cooperation with the Canadians. By mutual agreement, Station 30 + 00, the center point of the structure and the International Boundary was established as an exact point and all measurements taken from there.

Reference points for the centerline were set on the downstream cofferdam which was out of the way of most construction activity and above the work area in the early stages. Original cross sections were taken normal to the centerline by conventional methods.

For the first four months, quantity surveys for partial payment estimates were made by photogrammetric methods. Then the excavation was to ledge rock and more accurate work was necessary. Final cross sections for earth excavation became original sections for rock. By this time, the contractor had assembled a larger crew of surveyors and cross section work could be fitted in with other duties. As the excavation progressed to the point where accurate control was necessary to carve the rock to fit the requirements of the structure, control was transferred to each block area by transit lines. The control was referenced to block lines rather than center line of units for purposes of excavation and concrete form work.

With the start of concrete work, the establishment and protection of block control brought different problems. It is not easy to see into a block surrounded by forms and filled with all sorts of embedded items. The contractor's office engineering division prepared detail sheets for each pour showing form lines and locations of all embedded items. Field engineering determined the best way to bring in and the best places to set control points for the pour. These points were located on a layout sheet, superimposed on the detail sheets and prints run off on a small Ozalid machine in the field office for use by the various crafts.

When the time came to start setting units, their centerline had to be established from the original reference line. This center line is maintained on an independently supported base for a Wilde T2 theodolite. Vertical readings on units are made with a Zeiss precise level reading micrometer targets.

At Barnhart the contractor's surveying force reached a maximum of 40 men in eleven parties with generally three men on a party, assisted by the craft requiring the surveying.

Surveys for other major structures follow approximately the same general pattern. For Seaway work, the Corps of Engineers set several points along the centerline of locks, dikes, and the canal as well as bench marks. It also took original and final cross sections for excavation. The Canadians do all "engineering" work connected with construction contracts, including all surveying.

Channels

Surveying and mapping for channel work involved several interesting problems, all of which had been encountered in the past. The license for the project required very close regulation of water levels from Lake Ontario to Lake St. Francis. In order to program all work, a model was constructed by Ontario Hydro. There was sufficient data available from old records of soundings for all of the river except the Long Sault Rapids. Here the current was too fast for sounding from a boat and the river too wide to sound from a wire. Ontario Hydro took some 900 spot soundings from helicopters during

the period from October 1953 to June 1954 following the same method as devised by the Corps of Engineers for the Cascade section of the Niagara River in 1950. Soundings were taken from an elevation of less than 1000 feet. It was found necessary to employ a large Army helicopter because of the drafts above the rapids. Smaller crafts were unable to maintain position under varying wind velocities.

After design and programming had been completed, contracts were let by the Power Authority and Ontario Hydro for channel excavation and spoil. Contractors were required to do most of the surveying, being given monumented base lines, roughly parallel to the work, and bench marks. The Power Authority and Ontario Hydro furnished sonic depth recorders for all underwater measurements. This equipment with one operator furnished by the Authority was available for use by the contractors without charge. The contractor provided the other members of the survey crew and the boat. For purposes of measurement for payment, this echo sounding equipment is considered a precision instrument. One on each side of the international boundary was sufficient for all contracts. The Power Authority employed an Edo machine while Ontario Hydro used a Kelvin Hughes. On the American side, horizontal location was determined by keeping the boat on a range line by sighting over targets and measuring distance from shore by means of a wire measuring device developed by the Tennessee Valley Authority and mounted on the sounding boat. In fast water, the Americans, and in all water, the Canadians determined horizontal location by a fix from two transit lines.

The sounding equipment was furnished to the contractors without charge at all times. He was required to pay for boat and crew for other than original and final cross section work.

From the given data, contractors' surveyors established grid systems with range lines laid out at even stations and normal to the base line. Temporary bench marks were set throughout the areas on circuits run from the given permanent benches. Cross sections were taken by the use of steel tape on land and as previously mentioned for work under water.

Highway & Railroad

For highway and railroad relocation work, the Power Authority ran preliminary centerline and profiles and took cross sections to determine quantities of excavation and embankment. Contractors were required to re-run the centerlines between fixed P.I.s and do all other surveying necessary to complete the work and determine quantities for payment. All survey methods used were conventional.

Reservoir Clearing

The reservoir is being cleared generally to elevation 246. On steep high banks, the clearing is extended to a point where a two on one slope from the present water level would intersect the ground level. In flat country and areas designated for special use, the clearing line is set at 100 feet from the anticipated high operating water level.

From the topographic maps plotted from aerial photographs, it was possible to determine generally where the clearing line should be. From the

previously established horizontal and vertical control net, survey parties were able to stake out the clearing line easily in the areas of open to lightly wooded ground. In heavily wooded areas, it is difficult to see during the summer and insects are vicious. In the winter the snow is too deep. In the spring the mud and water is a problem. That leaves a short time in the fall during which surveying can be carried on efficiently. Some two thousand acres on the American side lies within a foot or so of the proposed clearing elevation. This area is covered with little ridges and valleys and is generally swampy.

The clearing line was staked out at intervals of 200 feet generally and tied in to physical features so that it could be shown on topographic maps. From aerial photographs and land reconnaissance, all areas to be cleared were classified as to type of cover and this classification, heavy, medium, sparse or open, was indicated on the maps which were furnished to prospective contractors for bidding purposes. Clearing contracts were let on a lump sum basis by schedules so that no further surveying is needed unless changes are made in the areas to be cleared under a given schedule.

Land Acquisition

All land necessary for the Power and Seaway Projects on the American side was acquired by the Power Authority using the facilities of the State of New York represented mainly by the State Department of Public Works and the Attorney General's Office.

As soon as designs were sufficiently firm in any area to indicate the approximate taking line, surveyors tied this line in to existing property monuments. Most of these were, of course, located by the old Ross Survey. Boundaries of individual holdings were re-run so that land parcel maps could be prepared. Quite often, the field measurements did not check those given on maps. After many months, it was discovered that in "the old days" established roads were considered community property and therefore the distance across a road was not included in any measurement between corners.

Parcel maps were prepared for all land to be taken. These referred to the legal description. Then all maps were converted to Canadian coordinates and plotted on topographic maps at a scale of 1 inch equals 500 feet.

Townships in Ontario are supposedly 9 miles by 12 miles. The original surveys of lands within the project area were run by different people using a variety of standards. The established line in the field do not agree with the recorded notes. North-south lines are generally parallel. East-west lines are not parallel and in many cases not straight. The variation from a straight line is a matter of miles in some instances.

Ontario Hydro had the problem of relocating several towns. All lot lines had to be established accurately by an "Ontario Land Surveyor" in accordance with the statutes of the Province of Ontario. In development of the townsites, lot monuments were destroyed many times, adding to the survey workload.

CONCLUSION

Throughout the surveying work for the project by all agencies, a major problem was and still is a lack of qualified personnel. After discussing the

matter with various agencies and with many of the surveyors, this lack is attributed to the fact that surveying is apparently not recognized sufficiently as a profession nor is the compensation considered to be on a level with the responsibility.

Successful accomplishment of this high and varied program is attributable to the complete cooperation of the many American and Canadian agencies involved in developing the power and transportation potentials of the St. Lawrence River.

Journal of the
SURVEYING AND MAPPING DIVISION
Proceedings of the American Society of Civil Engineers

A METEOROLOGICAL METHOD FOR PROFILE SURVEYING

L. V. Toralballo[1]
(Proc. Paper 1447)

SYNOPSIS

This paper derives a hydrodynamic equation similar to that which underlies the method generally known as Airborne Profile Recording but one that is free of the assumption that the wind is geostrophic. An aircraft is made to fly along an isobar and certain atmospheric measurements are taken enroute as well as at the ends of the trajectory and on the two ground points.

INTRODUCTION

The problem of determining the difference in elevation between two points on the earth's surface is a problem of considerable engineering importance. Various procedures for solving it are well known to surveyors and engineers. Most of these procedures make use of the level. A drawback of these methods is the fact that they are rather time consuming when the two points are far apart, say, fifty miles apart.

There is also a method based on the principle of the water manometer. A drawback of this is the high cost and unwieldiness of the long flexible tube.

More recently a method has been developed which makes use of an aircraft which is made to fly over a constant air pressure trajectory whose extremities are respectively over the two points, whose difference in elevation is sought. Photographs are taken from the aircraft along the route. The ground clearances at the extremities are determined by radar. There exists an approximate formula relating these measurements to the difference in geopotential between the two ground points. This method has recently been of quite wide use. It goes by the name of "airborne profile recording."

The above-mentioned formula makes the assumption that the wind is geostrophic. When the actual wind is far from being geostrophic, this formula may lead to considerable error.

In the present paper, we derive a formula which is valid whether or not the

Note: Discussion open until April 1, 1958. Paper 1447 is part of the copyrighted Journal of the Surveying and Mapping Division of the American Society of Civil Engineers, Vol. 83, No. SU 2, November, 1957.

1. Prof., College of Eng., New York Univ., New York, N. Y.

wind is geostrophic. Since the method requires the measurement of a large number of meteorological quantities, we have called it The Meteorological Method.

The Physical Principle Involved

The principle underlying this method may be described in the following manner:

An atmospheric curve, or trajectory, of constant pressure is a dynamical system in which wind velocity and a number of other atmospheric attributes, as they vary along the trajectory, are related to the gravitational potential as it varies along the same trajectory. Hence, by taking account of the various atmospheric quantities, one could expect to make a determination of the variation in gravitational potential. In point of fact, the constancy of the atmospheric pressure along the trajectory makes it possible to derive an explicit expression for the geopotential difference in terms of the air density, the wind velocity vector, and a number of derived kinematical quantities. As the variation in g (the acceleration of gravity) is small in the situations under consideration, this yields an expression for the difference in elevation.

General Description

Let A and B be the two points on the earth's surface whose difference in elevation is sought. An aircraft, carrying suitable instruments, flies along a constant pressure trajectory, which starts from a point C directly above point A and at least one thousand feet above it, and ends at a point E directly above point B. This trajectory is one for which the atmospheric pressure at a given point, at the time of arrival of the aircraft at that point, is the same as the atmospheric pressure at every other point at the corresponding time of arrival of the aircraft at that point. We call such a trajectory, a time isobar, or more simply, an isobar.

As the aircraft flies along the isobar, certain measurements are taken by means of instruments carried aboard. Other measurements are taken by means of instruments located on the ground, at the two stations whose difference in elevation is sought.

The measurements fall under the following classification:

a. Measurements effected by instruments carried aboard.

a.1. Measurements at the two ends of the trajectory.

a.1.a. The geometric heights of the ends of the trajectory with respect to the terrain. These are determined by radar altimeter.

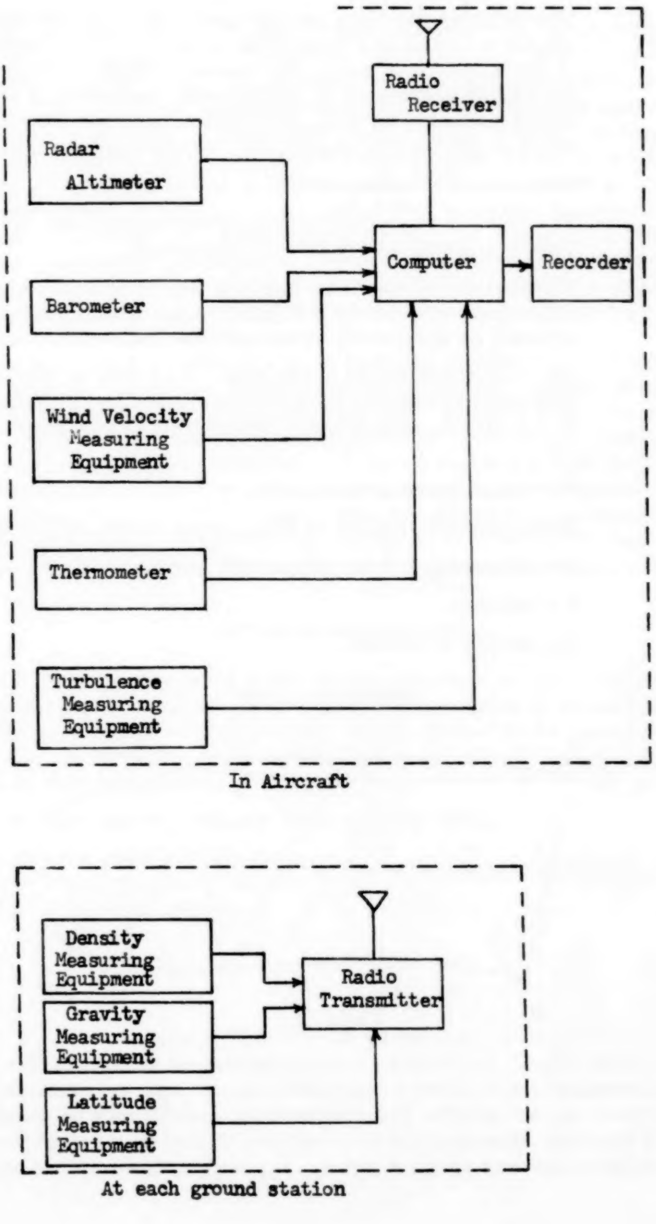
a.1.b. The temperature of the air.

a.1.c. The pressure of the air.

a.1.d. The height tendency of the isobaric surface. This is defined as the time rate of vertical motion of the isobaric surface. This is determined thus: The aircraft should fly a small circular course about C, the beginning of the trajectory, remaining all the time on the isobaric surface, and sufficiently close to C that the differences in elevation of the terrain directly below

The Meteorological Method for
Determining Difference in Elevation

Block Diagram



are close to zero. The altitude of the craft, with respect to this terrain, is measured at frequent intervals by radar altimeter. From these data, one may obtain an approximate value for the height tendency at the beginning of the trajectory. Similar measurements are taken at the end of the trajectory.

A.1.e. The pressure tendency, or the time rate of change of the atmospheric pressure at a fixed point of space. This is determined in the following manner: The aircraft should fly a small circular course about point C, all the while remaining at a constant height above the terrain. The atmospheric pressure is measured at frequent intervals, and recorded against time.

a.2. Measurements taken enroute.

a.2.a. The velocity vector of the wind with respect to the aircraft. This is effected by suitable anemometers.

a.2.b. The velocity vector of the aircraft with respect to the ground. This can be effected by a doppler radar system, or by a system utilizing an integrating horizontal accelerometer.

a.2.c. The turbulence stress of the wind. This may be measured by instrumental systems such as those described in Reference No. 53-52, Marine Meteorology, Woods Hole Oceanographic Institution.

a.2.d. The temperature of the air.

b. Measurements effected by instruments on the ground stations.

b.1 The value of g, the acceleration of gravity.

b.2 The latitude.

b.3 The density of the air.

System Analysis

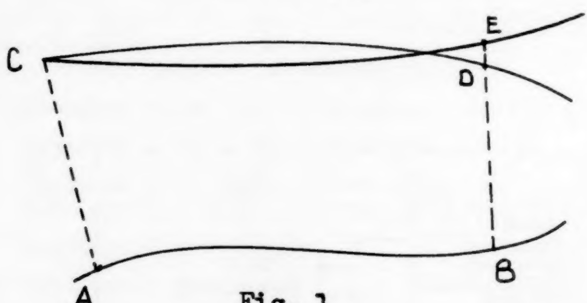


Fig. 1

Let A and B, Fig. 1, be the two ground points whose difference in elevation is to be determined. Let CD be a level line (i.e., an equi-gravitational potential line) above the two points. The distances AC and BD may be measured accurately by radar altimeter and their difference may be taken as the difference in elevation between points A and B. In practice, however, we have no

way of identifying level lines, or points on the same level line. It is simple to identify isobaric lines, (i.e., lines of constant atmospheric pressure). Let CE be such an isobaric line. Again we have no way of measuring the distance DE directly. However, there is a feasible way of determining this distance; this, essentially, consists in computing the slopes of the isobar CE with respect to the level line CD.

The air in the isobaric line CE lies simultaneously in two force fields. These are the gravitational field and the fluid pressure field associated with the velocity field of the air. This velocity field is a function of the rotational velocity of the earth, the latitude of the point and the overall temperature field of the atmosphere. If the airflow were strictly laminar the fundamental relationship between the resultant of the gravitational field and the fluid pressure field, on the one hand, and the velocity field on the other hand, are expressed mathematically by the so-called Navier-Stokes hydrodynamic partial differential equations. The fact that the airflow is generally considerably turbulent complicates the situation immensely. However, the new relationship can still be represented to a high degree of approximation by a modification of the Navier-Stokes equations which contain certain additional terms, the so-called Reynolds stress terms.

It has been found possible to derive from the modified Navier-Stokes equations a relation between the geopotential at a point and certain space-time quantities which obtain along the isobaric line. From this relation one may obtain an explicit expression for the slope of the isobaric line with respect to the corresponding geopotential line. We have found it possible to derive directly a relation between the difference in elevation and the space-time quantities mentioned above. These space-time quantities may be measured and recorded by suitable instruments carried by an aircraft which flies along the isobaric line, plus some additional information from ground stations.

Mathematical Analysis

Let P_o, Fig. 2, be a point in the earth's atmosphere. At P set up a tri-rectangular coordinate system (rotating with the earth) in which x-axis points East, the x_2-axis points North and x_3 axis is normal to the geodesic surface. Let v_1, v_2 and v_3 be the components of the wind velocity vector along the x_1, $x2$ and x_3 axes respectively.

Let W = the angular velocity of the rotating earth

Let ρ = the density of the air)
Let P = the pressure of the air) at the point whose coordinates are x_1, x_2, x_3
Let L = the local latitude)

$$\lambda = 2W \sin L$$

$$\alpha = \frac{1}{\rho}$$

The Eulerian equations of motion of the air may be expressed by the so-called Navier-Stokes equations, viz

Mathematical Analysis

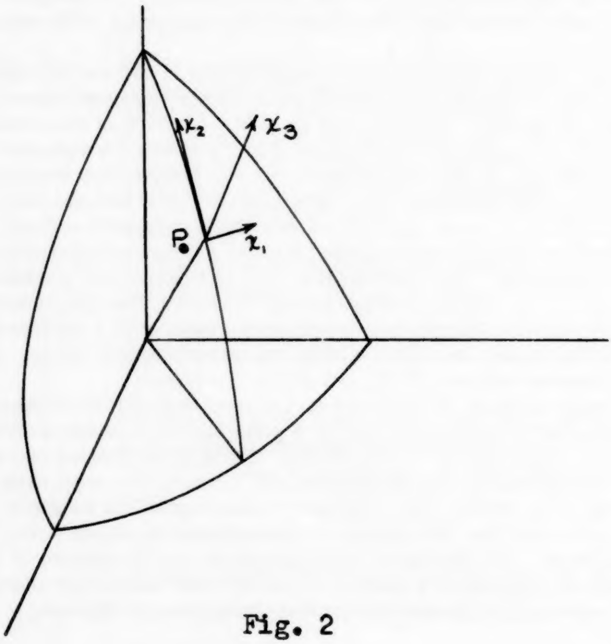


Fig. 2

$$\frac{dv_1}{dt} + (2\,W\cos L)v_3 - (2\,W\sin L)v_2 = -\alpha\,\frac{\partial P}{\partial x_1}$$

$$\frac{dv_2}{dt} + (2\,W\sin L)\,v_1 = -\alpha\,\frac{\partial P}{\partial x_2}$$

$$\frac{dv_3}{dt} - (2\,W\cos L)\,v_1 = -\alpha\,\frac{\partial P}{\partial x_3} - g$$

These equations refer to instantaneous values of the wind velocity components. Instruments for measuring velocity components necessarily read average values, the time span for the averaging depending upon the characteristics of the particular instrument. When the windflow is laminar the difference between the instantaneous values required by the equations and the average values as read by low-drag instruments presently available, is sufficiently small so that the above equations, with instrumental values inserted, represent the situation adequately.

However, the windflow in the atmosphere is seldom close to laminar. On this account, the above equations should be modified in such a manner as to make them applicable when the quantities entered are average values. This

is done in the following manner: A fluctuation, eddy, or turbulence, velocity, v_i', is defined as the difference between the instantaneous velocity v_i and the average velocity $\overline{v_i}$. Thus

$$v_i = \overline{v_i} + v_i'$$

$$\frac{dv_i}{dt} = \frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j}$$

using the tensor notation in which a given index used twice in a term denotes a summation with respect to that index. If the variations in the air density are negligible the equation of continuity becomes

$$\frac{\partial v_j}{\partial x_j} = 0$$

Hence

$$\frac{dv_i}{dt} = \frac{\partial v_i}{\partial t} + v_i \frac{\partial v_j}{\partial x_j} + v_j \frac{\partial v_i}{\partial x_j} = \frac{\partial v_i}{\partial t} + \frac{\partial v_i v_j}{\partial x_j}$$

Now

$$v_i = \overline{v_i} + v_i'$$

then

$$\frac{d\overline{v_i}}{dt} = \frac{\partial \overline{v_i}}{\partial t} + \frac{\partial \overline{v_i'}}{\partial t} + \frac{\partial}{\partial x_j} \left(\overline{\overline{v_i}\,\overline{v_j}} + \overline{\overline{v_i}\,v_j'} + \overline{\overline{v_j}\,v_i'} + \overline{v_i'\,v_j'} \right)$$

We now make the usual assumption that

$$\overline{\overline{v_i}\,\overline{v_j}} = \overline{v_i}\,\overline{v_j}$$
$$\overline{\overline{v_i}\,v_j'} = \overline{v_i}\,\overline{v_j'} = 0$$
$$\overline{\overline{v_j}\,v_i'} = \overline{v_j}\,\overline{v_i'} = 0$$

Hence

$$\frac{d\overline{v_i}}{dt} = \frac{\partial \overline{v_i}}{\partial t} + \overline{v_j} \frac{\partial \overline{v_i}}{\partial x_j} + \frac{\partial \overline{v_i' v_j'}}{\partial x_j}$$

When one compares this expression with the previous, unaveraged expression

$$\frac{dv_i}{dt} = \frac{\partial v_i}{\partial t} + V_j \frac{\partial v_i}{\partial x_j} ,$$

one notices that the use of average values leads to the addition of a new term,

$$\frac{\partial \overline{v_i' v_j'}}{\partial x_j}$$

Incorporating this result into the Navier-Stokes equations one obtains the modified forms of these equations:

$$\frac{d\overline{v_1}}{dt} + (2\,W\cos L)\overline{v_3} - (2\,W\sin L)\overline{v_2} = -\alpha\frac{\partial \overline{P}}{\partial x_1} - \frac{\partial \overline{v_1' v_K'}}{\partial x_K}$$

$$\frac{d\overline{v_2}}{dt} + (2\,W\sin L)\overline{v_1} = -\alpha\frac{\partial \overline{P}}{\partial x_2} - \frac{\partial \overline{v_2' v_K'}}{\partial x_K}$$

$$\frac{d\overline{v_3}}{dt} - (2\,W\cos L)\overline{v_1} = -\alpha\frac{\partial \overline{P}}{\partial x_3} - g - \frac{\partial \overline{v_3' v_K'}}{\partial x_K}$$

We shall subject the above set of equations to certain transformations and combinations in order to obtain the forms desired. For simplicity, we shall omit the bars on $\overline{v_1}$, $\overline{v_2}$, $\overline{v_3}$, $\overline{P}$. The first two of the above equations yield

$$\frac{d}{dt}(v_1 + i v_2) + i\lambda(v_1 + i v_2) = -\alpha\left(\frac{\partial P}{\partial x_1} + i\frac{\partial P}{\partial x_2}\right) - (2\,W\cos L)v_3 - \left(\frac{\partial \overline{v_1' v_K'}}{\partial x_K} + i\,\frac{\partial \overline{v_2' v_K'}}{\partial x_K}\right)$$

$$v_1 + i v_2 = \frac{\alpha i}{\lambda}\left(\frac{\partial P}{\partial x_1} + i\,\frac{\partial P}{\partial x_2}\right) + \frac{i}{\lambda}\,\frac{d}{dt}(v_1 + i v_2) + \frac{i}{\lambda}(2\,W\cos L)v_3 + \frac{i}{\lambda}\left(\frac{\partial \overline{v_1' v_K'}}{\partial x_K} + i\,\frac{\partial \overline{v_2' v_K'}}{\partial x_K}\right)$$

Now,

$$\frac{d}{dt}(v_1 + i v_2) = \frac{\partial}{\partial t}(v_1 + i v_2) + \left(v_1\frac{\partial}{\partial x_1} + v_2\frac{\partial}{\partial x_2}\right)(v_1 + i v_2)$$

Hence

$$v_1 + i v_2 = \frac{i}{\lambda}\alpha\left(\frac{\partial P}{\partial x_1} + i\frac{\partial P}{\partial x_2}\right) + \frac{i}{\lambda}\left\{\frac{\partial}{\partial t}(v_1 + i v_2) + \left(v_1\frac{\partial}{\partial x_1} + v_2\frac{\partial}{\partial x_2}\right)(v_1 + i v_2) + (2\,W\cos L)v_3 + \frac{\partial}{\partial x_K}\left(\overline{v_1' v_K'} + i\,\overline{v_2' v_K'}\right)\right\}$$

We shall write $v_1 + i\, v_2$ as the sum of two terms

$v_1 + i\, v_2 = W + J$, where

$$W = \frac{\alpha i}{\lambda}\left(\frac{\partial P}{\partial x_1} + i\frac{\partial P}{\partial x_2}\right) \qquad \text{and}$$

$$J = \frac{i}{\lambda}\left\{\frac{\partial}{\partial t}\,(v_1 + i v_2) + \left(v_1\frac{\partial}{\partial x_1} + v_2\frac{\partial}{\partial x_2}\right)(v_1 + i v_2) + (2W\cos L)v_3 + \frac{\partial}{\partial x_K}\left(\overline{v_1' v_K'} + i\,\overline{v_2' v_K'}\right)\right\}$$

We now consider the components of W and J at right angles to the track of the aircraft. We shall refer to Fig. 3. Let the component of W at right angles to the track be denoted by W*. We now evaluate W*.

$$P + \left(\frac{\partial P}{\partial x_3}\right)\Delta x_3 + \left(\frac{\partial P}{\partial x_2}\right)\Delta x_2 + \left(\frac{\partial P}{\partial x_1}\right)\Delta x_1 + \left(\frac{\partial P}{\partial t}\right)\Delta t = P$$

$$\therefore \frac{\partial P}{\partial x_1}\frac{dx_1}{ds} + \frac{\partial P}{\partial x_2}\frac{dx_2}{ds} = -\frac{\partial P}{\partial x_3}\frac{dx_3}{ds} - \frac{\partial P/\partial t}{G}$$

where $G = \dfrac{ds}{dt}$

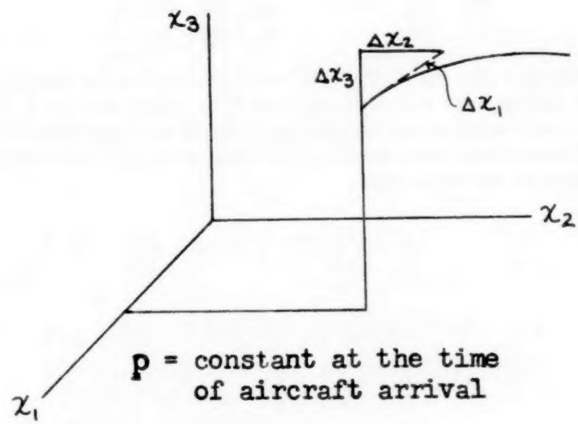


Fig. 3

Let OP, in Fig. 4, be tangent to the tract at 0. The component of $-\frac{\alpha}{\lambda}\frac{\partial P}{\partial x_2}$ at right angles to the track is $\frac{-\alpha}{\lambda}\frac{\partial P}{\partial x_2}\sin\tau$; that of $\frac{\alpha}{\lambda}\frac{\partial P}{\partial x_1}$ is $\frac{\alpha}{\lambda}\frac{\partial P}{\partial x_1}\sin\eta$. The total component of W at right angles to the track is

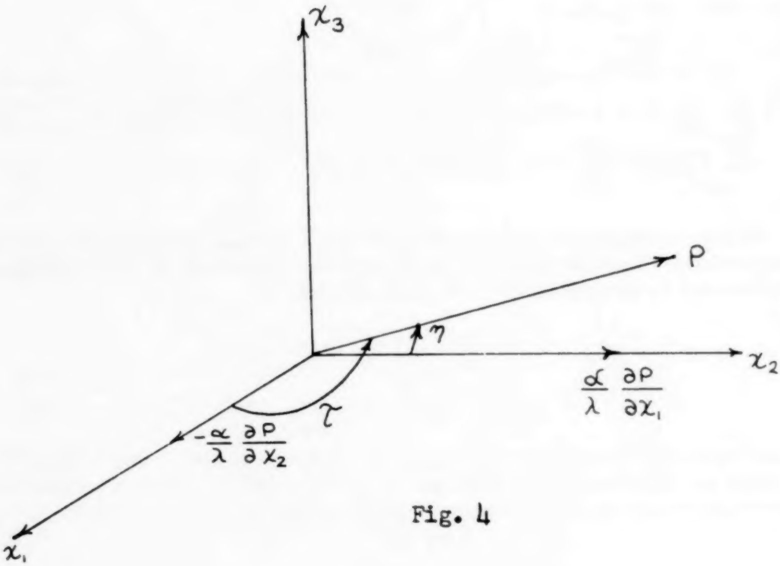


Fig. 4

$$-\frac{\alpha}{\lambda}\frac{\partial P}{\partial x_2}\sin\tau - \frac{\alpha}{\lambda}\frac{\partial P}{\partial x_1}\sin\eta$$

In view of the fact that the slope of OP with respect to the horizontal plane is quite small and the fact that replacing sin τ by cos η and sin η by cos τ tend to increase both the minuend and the subtrahend and thus these replacements are almost completely compensating, we may write the total component of W at right angles to the track thus:

$$\begin{aligned} W^* &= -\frac{\alpha}{\lambda}\frac{\partial P}{\partial x_2}\cos\eta - \frac{\alpha}{\lambda}\frac{\partial P}{\partial x_1}\cos\tau \\ &= -\frac{\alpha}{\lambda}\left(\frac{\partial P}{\partial x_2}\frac{dx_2}{ds} + \frac{\partial P}{\partial x_1}\frac{dx_1}{ds}\right) \\ &= -\frac{\alpha}{\lambda}\left(-\frac{\partial P}{\partial x_3}\frac{dx_3}{ds} - \frac{\partial P/\partial t}{G}\right) \\ &= \frac{\alpha}{\lambda}\left(\frac{\partial P}{\partial x_3}\frac{dx_3}{ds} + \frac{\partial P/\partial t}{G}\right) \end{aligned}$$

We shall now use the third equation of motion.

$$\frac{dv_3}{dt} - (2W\cos L)v_1 + g = -\alpha\frac{\partial P}{\partial x_3} - \frac{\partial\overline{v_3' v_k'}}{\partial x_k}$$

$$\frac{\partial P}{\partial x_3} = -\frac{1}{\alpha}\left(\frac{dv_3}{dt} - (2W\cos L)v_1 + g + \frac{\partial\overline{v_3' v_k'}}{\partial x_k}\right)$$

Hence

$$W^{*} = \frac{1}{\lambda}\left[-\frac{dv_3}{dt} + (2W\cos L)v_1 - \frac{\partial\overline{v_3 v_K}}{\partial x_K} - g\right]\frac{dx_3}{ds} + \frac{\alpha}{\lambda}\,\frac{\partial P/\partial t}{G}$$

Now $g = \dfrac{\partial\Phi}{\partial x_3}$, where Φ is the geopotential, hence

$$W^{*} = \frac{1}{\lambda}\left[-\frac{dv_3}{dt} + (2W\cos L)v_1 - \frac{\partial\overline{v_3' v_K'}}{\partial x_K} - \frac{\partial\Phi}{\partial x_3}\right]\frac{dx_3}{ds} + \frac{\alpha}{\lambda}\,\frac{\partial P/\partial t}{G}$$

We now consider the component of J at right angles to the track of the aircraft. We designate it by J*. We now refer to Figs. 5 and 6.

$$J = \frac{i}{\lambda}\left[\frac{\partial}{\partial t}(v_1 + i v_2) + v_1\frac{\partial}{\partial x_1} + v_2\frac{\partial}{\partial x_2}\right)(v_1 + i v_2) + (2W\cos L)v_3$$

$$+ \frac{\partial}{\partial x_K}\left(\overline{v_1' v_K'} + i\,\overline{v_2' v_K'}\right)\Big]$$

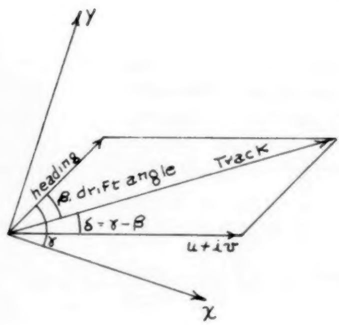


Fig. 5

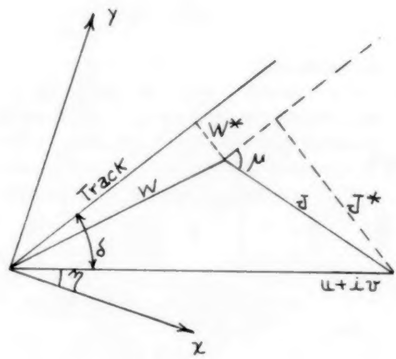


Fig. 6

A gyroscopic device maintains the direction of the x-axis. Let

η = the argument of the total horizontal wind.
Let θ = the argument of J
Let $\mu = \eta + \delta - \theta$

Then

$J^{*} = /J/ \sin\mu$
Let A = the absolute value of the horizontal wind velocity.

Then $A\sin\delta = W^{*} + J^{*}$
$W^{*} = A\sin\delta - J^{*}$

Hence

$$A \sin\delta - J^* = \frac{1}{\lambda}\left[\frac{dv_3}{dt} + (2W\cos L)v_1 - \frac{\partial\overline{v_3'v_k'}}{\partial x_k} - \frac{\partial\phi}{\partial x_3}\right]\frac{dx_3}{ds} + \frac{\alpha}{\lambda}\left(\frac{\partial p/\partial t}{G}\right)$$

$$A\sin\delta - J^* = \frac{1}{\lambda}\left[-\frac{dv_3}{dt} + (2W\cos L)v_1 - \frac{\partial\overline{v_3'v_k'}}{\partial x_k}\right]\frac{dx_3}{ds}$$

$$+ \frac{\alpha}{\lambda}\frac{\partial P/\partial t}{G} - \frac{1}{\lambda}\frac{\partial\Phi}{\partial s}$$

$$\frac{\partial\Phi}{\partial s} = \left[-\frac{dv_3}{dt} + (2W\cos L)v_1 - \frac{\partial\overline{v_3'v_k'}}{\partial x_k}\right]\frac{dx_3}{ds}$$

$$- \lambda(A\sin\delta - J^*) + \alpha\left(\frac{\partial P/\partial t}{G}\right)$$

Now, the airplane flies at a constant pressure.

$$\therefore \frac{d\Phi}{dt} = \left(\frac{\partial\Phi}{\partial t}\right)_{P,S} + \left(\frac{\partial\Phi}{\partial s}\right)_{P,t}\frac{ds}{dt}$$

$$\frac{d\Phi}{dt} = g\left(\frac{\partial x_3}{\partial t}\right)_{P,S} + \left(\frac{\partial\Phi}{\partial s}\right)_{P,t}\frac{ds}{dt}$$

Hence

$$\Phi_2 - \Phi_1 = \int_{t_1}^{t_2} g\left(\frac{\partial x_3}{\partial t}\right)_{P,S} dt + \int_{S_1}^{S_2}\left\{\left[-\frac{dv_3}{dt} + (2W\cos L)v_1\right.\right.$$

$$\left.\left. - \frac{\partial\overline{v_3'v_k'}}{\partial x_k}\right]\frac{dx_3/dt}{G} - \lambda(A\sin\delta - J^*) + \alpha\left(\frac{\partial P/\partial t}{G}\right)\right\}ds$$

in which Φ_1, and Φ_2 are respectively, the geopotentials at the beginning and at the end of the isobaric trajectory and $\left.\frac{\partial x_3}{\partial t}\right)_{P,S}$ is the height tendency of the isobaric surface.

Let z_1 and z_2 be the heights of the ends of the isobaric trajectory with respect to sea level.

$$z_2 - z_1 = \frac{\Phi_2}{g_2} - \frac{\Phi_1}{g_1} \equiv \frac{\frac{g_1+g_2}{2}(\Phi_2 - \Phi_1) + (g_1 - g_2)\left(\frac{\Phi_1 + \Phi_2}{2}\right)}{g_1 g_2}$$

We assume that $g_1 - g_2$ is so small that the contribution of

$\frac{g_1 - g_2}{g_1 g_2}\;\frac{\Phi_1 + \Phi_2}{2}$ to $z_2 - z_1$ can be neglected.

Let $\frac{g_1 + g_2}{2} = \overline{g}$ and assume that $g_1 g_2 \doteq \overline{g}^2$ then

$z_2 - z_1 = \frac{\Phi_2 - \Phi_1}{\overline{g}}$, approximately.

Let H_1 and H_2 be the respective altitudes, with respect to sea level of the two points whose difference in elevation is sought. Let h_1 and h_2 be the respective altitudes of the aircraft at the ends of the trajectory, with respect to the terrain.

Then $z_1 = H_1 + h_1$ and $z_2 = H_2 + h_2$

$H_2 - H_1 = (h_1 - h_2) + (z_2 - z_1)$

h_1 and h_2 are to be determined by radar.

Thus

$$H_2 - H_1 = (h_1 - h_2) + \int_{t_1}^{t_2} \left(\frac{\partial x_3}{\partial t}\right)_{P.S} dt + \frac{1}{\overline{g}}\left[\int_{s_1}^{s_2} \left\{ \left[\frac{-dv_3}{dt} + (2W\cos L) v_1 - \frac{\partial \overline{v_3' v_k'}}{\partial x_k}\right] \frac{\frac{dx_3}{dt}}{G} - \lambda (A \sin\delta - J^*) + \alpha \frac{\partial P / \partial t}{G} \right\} ds \right]$$

Corrections for the Deviation from the Isobaric Course

The method prescribes that the aircraft fly on an isobar. This is an important assumption in the development of the ideal equation. Needless to say, this is not completely feasible, and with equipment presently available, the actual path taken by the aircraft may differ considerably from the isobaric course. In practice, the aircraft is bound to weave through the isobar. It is therefore of importance to compute a correction for this deviation.

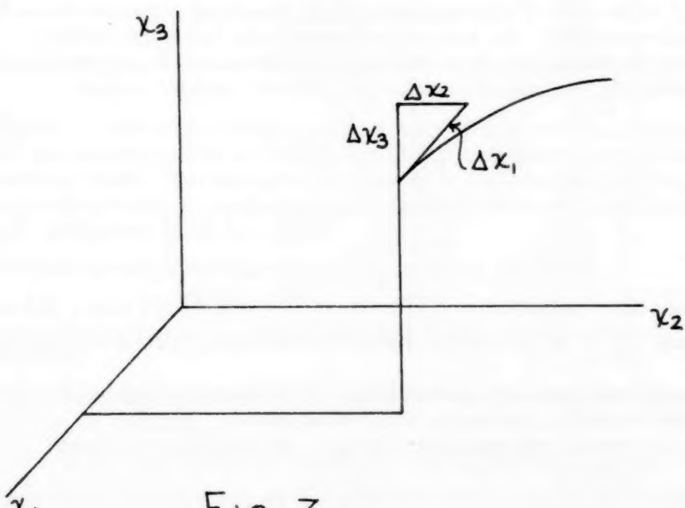


Fig. 7

$$P + \left(\frac{\partial P}{\partial x_3}\right)\Delta x_3 + \left(\frac{\partial P}{\partial x_2}\right)\Delta x_2 + \left(\frac{\partial P}{\partial x_1}\right)\Delta x_1 + \left(\frac{\partial P}{\partial t}\right)\Delta t = P + \Delta P$$

$$\therefore \frac{\partial P}{\partial x_1}\frac{dx_1}{ds} + \frac{\partial P}{\partial x_2}\frac{dx_2}{ds} = -\frac{\partial P}{\partial x_3}\frac{dx_3}{ds} - \frac{\partial P/\partial t}{G} + \frac{dp}{ds}$$

A new term, $\frac{dp}{ds}$, thus appears.

$$W^* = \frac{\alpha}{\lambda}\left[\frac{\partial P}{\partial x_3}\frac{dx_3}{ds} + \frac{\partial P/\partial t}{G} - \frac{dp}{ds}\right]$$

$$= \frac{1}{\lambda}\left[-\frac{dv_3}{dt} + (2W\cos L)v_1 - \frac{\partial \overline{v_3' v_k'}}{\partial x_k} - g\right]\frac{dx_3}{ds} + \frac{\alpha}{\lambda}\frac{\partial P/\partial t}{G} - \frac{\alpha}{\lambda}\frac{dp}{ds}$$

$$\frac{\partial \Phi}{\partial s} = \left[-\frac{dv_3}{dt} + (2W\cos L)v_1 - \frac{\partial \overline{v_3 v_k}}{\partial x_k}\right]\frac{dx_3}{ds} - \lambda(A\sin\delta - J^*)$$

$$+ \alpha\left(\frac{\partial P/\partial t}{G}\right) - \alpha\frac{dp}{ds}$$

The total derivative $\frac{d\Phi}{dt}$ may be expressed thus:

$$\frac{d\Phi}{dt} = \left(\frac{\partial \Phi}{\partial t}\right)_s + \left(\frac{\partial \Phi}{\partial s}\right)_t\frac{ds}{dt} = g\left(\frac{\partial x_3}{\partial t}\right)_s + \left(\frac{\partial \Phi}{\partial s}\right)_t\frac{ds}{dt}$$

We note that $\left(\frac{\partial x_3}{\partial t}\right)_s$ is the algebraic sum of two components, C_1 and C_2. The first is the height tendency $\left(\frac{\partial x_3}{\partial t}\right)_{P,s}$ of the isobaric surface, and the second is the rate of vertical deviation of the actual trajectory from the ideal isobaric trajectory. C_2 may be evaluated in the following manner:

Since the deviations from the isobaric path are of the nature of small perturbations, we may assume the hydrostatic equation to hold.

$$\frac{\partial P}{\partial x_3} = -g\rho$$

$$\frac{\partial P/\partial t}{\partial x_3/\partial t} = -g\rho$$

$$\therefore \frac{\partial x_3}{\partial t} = -\frac{1}{g}\alpha\left(\frac{\partial P}{\partial t}\right)_s$$

in which $\left(\frac{\partial P}{\partial t}\right)_s$ is the partial derivative obtainable from a continuous record of atmospheric pressure against time. This pressure may be recorded by a sensitive recording barometer carried on board.

The corresponding expression for $\phi_2 - \phi_1$ is, thus, as follows:

$$\Phi_2 - \Phi_1 = \int_{t_1}^{t_2} g\left(\frac{\partial x_3}{\partial t}\right)_{P,S} dt - \alpha \int_{t_1}^{t_2} \left(\frac{\partial P}{\partial t}\right)_S dt$$
$$+ \int_{S_1}^{S_2} \left\{ \left[-\frac{dv_3}{dt} + (2W\cos L)v_1 - \frac{\partial \overline{v_3' v_k'}}{\partial x_k} \right] \frac{\frac{dx_3}{dt}}{G} - \lambda(A\sin\delta - J^*) + \alpha\frac{\partial P/\partial t}{G} \right\} ds - (P_2 - P_1)\alpha$$

It is seen that there are two corrections due to deviations from the isobaric path. One is the terminal correction $-\alpha(P_2 - P_1)$, where p_1 and p_2 are the atmospheric pressures at the two ends of the trajectory. The second correction is an in-flight correction and is given by

$$-\alpha \int_{t_1}^{t_2} \left(\frac{\partial p}{\partial t}\right)_s dt$$

The Physical Assumptions

In the development of the above expressions for the difference in elevation, certain physical assumptions were made. In the main, these were as follows:

1. The variations in the value of g along the track are negligible or self-compensating.
2. The variations in the density of the air along the track are negligible or self-compensating.
3. The effects of the molecular viscosity of the air are negligible.

Instrumentation Required

The chief instruments required by the Meteorological Method are the following:

1. An airplane. This craft will be used both as a vehicle and as a meteorological instrument for the determination of the turbulence factors along the isobaric track. For the latter purpose it should have adequate instrumental systems such as those described in detail in Woods Hole Oceanographic Institution, Reference 52-53.
2. A precision barometer for keeping the aircraft on the isobar.
3. Recording vector anemomometer to record the magnitudes of the three components of the velocity of the wind, with respect to the aircraft, against time.
4. An instrument system to determine the three components of the ground velocity of the aircraft.
5. Instruments for measuring the latitudes of the two terminal points.
6. An instrument system, such as the APR, for measuring the drift angles of the aircraft.

7. An instrument for measuring the density of the air at the terminal points.
8. An instrument for measuring the acceleration of gravity at the terminal points.
9. A computer on board the craft.

CONCLUDING REMARKS

The fact that it is possible to express, at least approximately, the difference in elevation between two points in terms of the variation of wind velocity on an isobaric course was shown by T.J.C. Henry. Henry made the assumption that the wind is geostrophic. A number of organizations, both in the United States and Canada, have developed operational systems making use of Henry's formula. The accuracies obtained are, in general, in the order of fifteen feet.

Our expression for $H_2 - H_1$ was developed with a view to improving on this accuracy by obtaining a formula which is not subject to the restriction that the wind is geostrophic. In point of fact, our formula, subject to relatively broad physical assumptions, takes a more or less full account of all the various factors determinative of a difference in elevation. It would thus seem that our formula, within the approximations induced by the above-mentioned physical assumptions, should yield almost all the information contained in the Navier-Stokes aerodynamic equations.

As to the accuracies obtainable by the use of our formula, these will depend upon the instrumental accuracies with which the various atmospheric factors are measured. A source of considerable error is the relative lack of accuracy in the continuous determination of the velocity vector of the aircraft with respect to the ground, as the aircraft flies on its isobaric trajectory. The horizontal components of this vector, at the present state of art, are much more easily determined to prescribed accuracies than the vertical component. However, as the vertical component of the velocity of the wind is of the order of one per cent of the total horizontal component, a relatively large percentage error in the determination of the vertical component of the velocity of the aircraft will introduce only a relatively small error in the values given by our formula.

Paper 1448 November, 1957 SU 2

Journal of the
SURVEYING AND MAPPING DIVISION
Proceedings of the American Society of Civil Engineers

CONTENTS

DISCUSSION
(Proc. Paper 1448)

Page

Coordinated Surveying and Mapping for Industry, by E. D. Morse.
(Proc. Paper 1064. September, 1956. Prior discussion: 1319.
Discussion closed.)
by E. D. Morse (closure) 1448-3

COORDINATED SURVEYING AND MAPPING FOR INDUSTRY[a]

Closure by E. D. Morse

E. D. MORSE,* M. ASCE.—Mr. Mims, a practicing land surveyor of wide experience in Texas, has properly emphasized the economic obstacles to the employment of such modern surveying techniques as State Plane Coordinates by the private surveyor. This is particularly so in rural areas where land values may be low and there is either an absence or a relative scarcity of coordinated survey control points. Admittedly, tangible economic advantage is the soundest basis for the guidance of the surveyor or engineer, whether he is in pricate practice or employed by industry. Mr. Mim's point that the public in a given urban area may pay several times for duplicated and uncorrelated surveying and mapping efforts by various industries and local agencies is well taken.

Mr. Walston has presented an able discussion of the selective use of the State Plane Coordinate Systems from the standpoint of one of the major oil companies. His premise that the decision for such use in much of the oil industry, particularly in his own company, is dependent on economics rather than on a lack of trained personnel or an understanding of the advantages, can hardly be questioned. Certainly, several of the major oil companies, including Mr. Walston's, were among the first of the industries to employ coordinated surveying and mapping methods and they may well have made greater use of the State Plane Coordinate Systems than all other industries combined. It is worth noting, however, that some few of the oil companies find it advantageous to require the utilization of State Plane Coordinates whenever feasible whereas many others apparently do so only when it cannot be avoided, as is the case with offshore operations. Therefore, even when applied to the oil industry, there would appear to be some basis for the author's introductory statement to the effect that a good part of the reluctance of industry to adopt the use of State Plane Coordinates must stem from a mistaken idea of the complexity and cost relative to the lasting benefits that can be derived.

The author acknowledges his appreciation to the discussers for their constructive and informed treatment of this paper.

a. Proc. Paper 1064, September, 1957, by E. D. Morse.

* Supt. of Civ. Eng., Houston Lighting and Power Co., Houston, Tex.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Board of Direction are identified by the symbols (BD). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper numbers are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 1113 is identified as 1113 (HY6) which indicates that the paper is contained in the sixth issue of the Journal of the Hydraulics Division during 1956.

VOLUME 82 (1956)

NOVEMBER: 1096(ST6), 1097(ST6), 1098(ST6), 1099(ST6), 1100(ST6), 1101(ST6), 1102(IR3), 1103 (IR3), 1104(IR3), 1105(IR3), 1106(ST6), 1107(ST6), 1108(ST6), 1109(AT3), 1110(AT3)[c], 1111(IR3)[c]; 1112(ST6)[c].

DECEMBER: 1113(HY6), 1114(HY6), 1115(SA6), 1116(SA6), 1117(SU3), 1118(SU3), 1119(WW5), 1120(WW5), 1121(WW5), 1122(WW5), 1123(WW5), 1124(WW5)[c], 1125(BD1)[c], 1126(SA6), 1127 (SA6), 1128(WW5), 1129(SA6)[c], 1130(PO6)[c], 1131(HY6)[c], 1132(PO6), 1133(PO6), 1134(PO6), 1135(BD1).

VOLUME 83 (1957)

JANUARY: 1136(CP1), 1137(CP1), 1138(EM1), 1139(EM1), 1140(EM1), 1141(EM1), 1142(SM1), 1143(SM1), 1144(SM1), 1145(SM1), 1146(ST1), 1147(ST1), 1148(ST1), 1149(ST1), 1150(ST1), 1151(ST1), 1152(CP1)[c], 1153(HW1), 1154(EM1)[c], 1155(SM1)[c], 1156(ST1)[c], 1157(EM1), 1158 (EM1), 1159(SM1), 1160(SM1), 1161(SM1).

FEBRUARY: 1162(HY1), 1163(HY1), 1164(HY1), 1165(HY1), 1166(HY1), 1167(HY1), 1168(SA1), 1169(SA1), 1170(SA1), 1171(SA1), 1172(SA1), 1173(SA1), 1174(SA1), 1175(SA1), 1176(SA1), 1177(HY1)[c], 1178(SA1), 1179(SA1), 1180(SA1), 1181(SA1), 1182(PO1), 1183(PO1), 1184(PO1), 1185(PO1)[c].

MARCH: 1186(ST2), 1187(ST2), 1188(ST2), 1189(ST2), 1190(ST2), 1191(ST2), 1192(ST2)[c], 1193 (PL1), 1194(PL1), 1195(PL1).

APRIL: 1196(EM2), 1197(HY2), 1198(HY2), 1199(HY2), 1200(HY2), 1201(HY2), 1202(HY2), 1203 (SA2), 1204(SM2), 1205(SM2), 1206(SM2), 1207(SM2), 1208(WW1), 1209(WW1), 1210(WW1), 1211(WW1), 1212(EM2), 1213(EM2), 1214(EM2), 1215(PO2), 1216(PO2), 1217(PO2), 1218 (SA2), 1219(SA2), 1220(SA2), 1221(SA2), 1222(SA2), 1223(SA2), 1224(SA2), 1225(PO)[c], 1226 (WW1)[c], 1227(SA2)[c], 1228(SM2)[c], 1229(EM2)[c], 1230(HY2)[c].

MAY: 1231(ST3), 1232(ST3), 1233(ST3), 1234(ST3), 1235(IR1), 1236(IR1), 1237(WW2), 1238(WW2), 1239(WW2), 1240(WW2), 1241(WW2), 1242(WW2), 1243(WW2), 1244(HW2), 1245(HW2), 1246 (HW2), 1247(HW2), 1248(WW2), 1249(HW2), 1250(HW2), 1251(WW2), 1252(WW2), 1253(IR1), 1254(ST3), 1255(ST3), 1256(HW2), 1257(IR1)[c], 1258(HW2)[c], 1259(ST3)[c].

JUNE: 1260(HY3), 1261(HY3), 1262(HY3), 1263(HY3), 1264(HY3), 1265(HY3), 1266(HY3), 1267 (PO3), 1268(PO3), 1269(SA3), 1270(SA3), 1271(SA3), 1272(SA3), 1273(SA3), 1274(SA3), 1275 (SA3), 1276(SA3), 1277(HY3), 1278(HY3), 1279(PL2), 1280(PL2), 1281(PL2), 1282(SA3), 1283 (HY3)[c], 1284(PO3), 1285(PO3), 1286(PO3), 1287(PO3)[c], 1288(SA3)[c].

JULY: 1289(SM3), 1290(EM3), 1291(EM3), 1292(EM3), 1293(EM3), 1294(HW3), 1295(HW3), 1296(HW3), 1297(HW3), 1298(HW3), 1299(SM3), 1300(SM3), 1301(SM3), 1302(ST4), 1303 (ST4), 1304(ST4) 1305(SU1), 1306(SU1), 1307(SU1), 1308(ST4), 1309(SM3), 1310(SU1)[c], 1311(EM3)[c], 1312 (ST4), 1313(ST4), 1314(ST4), 1315(ST4), 1316(ST4), 1317(ST4), 1318 (ST4), 1319(SM3)[c], 1320(ST4), 1321(ST4), 1322(EM3), 1323(AT1), 1324(AT1), 1325(AT1), 1326(AT1), 1327(AT1), 1328(AT1)[c], 1329(ST4)[c].

AUGUST: 1330(HY4), 1331(HY4), 1332(HY4), 1333(SA4), 1334(SA4), 1335(SA4), 1336(SA4), 1337(SA4), 1338(SA4), 1339(CO1) 1340(CO1), 1341(CO1), 1342(CO1), 1343(CO1), 1344(PO4), 1345(HY4), 1346(PO4)[c], 1347(BD1), 1348(HY4)[c], 1349(SA4)[c], 1350(PO4), 1351(PO4).

SEPTEMBER: 1352(IR2), 1353(ST5), 1354(ST5), 1355(ST5), 1356(ST5), 1357(ST5), 1358(ST5), 1359(IR2), 1360(IR2), 1361(ST5), 1362(IR2), 1363(IR2), 1364(IR2), 1365(WW3), 1366(WW3), 1367(WW3), 1368(WW3), 1369(WW3), 1370(WW3), 1371(HW4), 1372(HW4), 1373(HW4), 1374(HW4), 1375(PL3), 1376(PL3), 1377(IR2)[c], 1378(HW4)[c], 1379(IR2), 1380(HW4), 1381(WW3)[c], 1382(ST5)[c], 1383(PL3)[c], 1384(IR2), 1385(HW4), 1386(HW4).

OCTOBER: 1387(CP2), 1388(CP2), 1389(EM4), 1390(EM4), 1391(HY5), 1392(HY5), 1393(HY5), 1394(HY5), 1395(HY5), 1396(PO5), 1397(PO5), 1398(PO5), 1399(EM4), 1400(SA5), 1401(HY5), 1402(HY5), 1403(HY5), 1404(HY5), 1405(HY5), 1406(HY5), 1407(SA5), 1408(SA5), 1409(SA5), 1410(SA5), 1411(SA5), 1412(EM4), 1413(EM4), 1414(PO5), 1415(EM4)[c], 1416(PO5)[c], 1417 (HY5)[c], 1418(EM4), 1419(PO5), 1420(PO5), 1421(PO5), 1422(SA5)[c], 1423(SA5), 1424(EM4), 1425(CP2).

NOVEMBER: 1426(SM4), 1427(SM4), 1428(SM4), 1429(SM4), 1430(SM4)[c], 1431(ST6), 1432 (ST6), 1433(ST6), 1434(ST6), 1435(ST6), 1436(ST6), 1437(ST6), 1438(SM4), 1439(SM4), 1440(ST6), 1441(ST6), 1442(ST6)[c], 1443(SU2), 1444(SU2), 1445(SU2), 1446(SU2), 1447 (SU2), 1448(SU2)[c].

---
c. Discussion of several papers, grouped by Divisions.